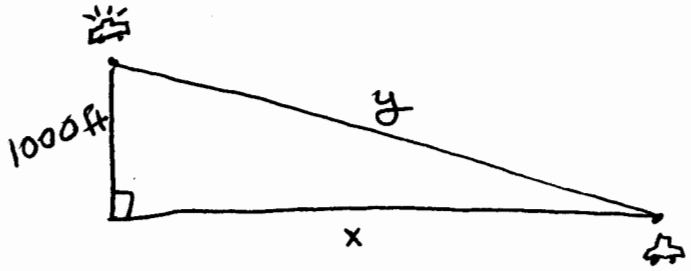
Related Rates Solutions

① 

At $t = 35$ sec,

$y = 1000$ ft

$\frac{dy}{dt} = 95$ ft/sec.

What is $\frac{dx}{dt}$?

$1000^2 + x^2 = y^2$ ⟶ When $y = 3000$:

$0 + 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$1000^2 + x^2 = 3000^2$

$x^2 = 8000000$

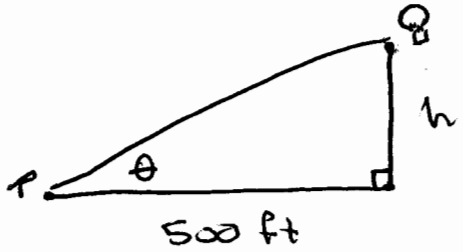
$x \approx 2828.4$ ft

$\frac{dx}{dt} = \frac{y\frac{dy}{dt}}{x}$

$= \frac{(3000)(95 \text{ ft/sec})}{2828 \text{ ft}} \approx 100.8$ ft/sec

$100.8 \frac{\text{ft}}{\text{sec}} \left(\frac{3600 \text{ sec}}{1 \text{ hr}}\right)\left(\frac{1 \text{ mi}}{5280 \text{ ft}}\right) \approx \boxed{68.7 \text{ mph}}$

You were speeding!

② 

When $\theta = \pi/4$ rad,

$\frac{d\theta}{dt} = .14$ rad/min.

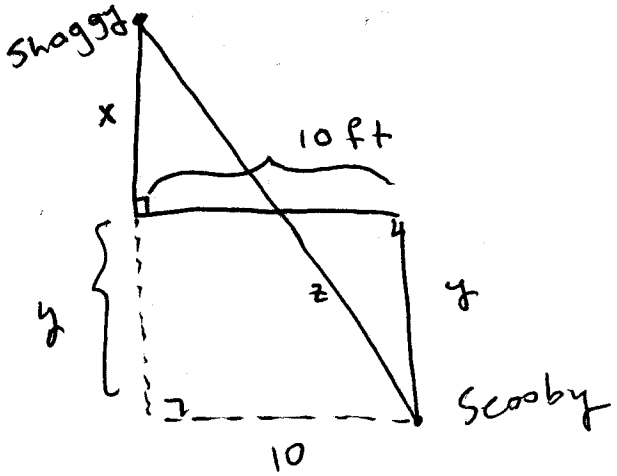
What is $\frac{dh}{dt}$?

$\tan\theta = \frac{h}{500} \Rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{500} \cdot \frac{dh}{dt}$

$$\Rightarrow \frac{dh}{dt} = 500\left(\sec\frac{\pi}{4}\right)^2(.14) = \boxed{140 \text{ ft/min}}$$

(3) See Example 3 on page 267.

(4)



When $t = 10$ s,

$\frac{dx}{dt} = 4$ ft/s

$\frac{dy}{dt} = 3$ ft/s.

What is $\frac{dz}{dt}$?

When $t = 10$ s, $x = 4 \text{ ft/s} \cdot 10 \text{ s} = 40$ ft

$y = 3 \text{ ft/s} \cdot 10 \text{ s} = 30$ ft

$(x+y)^2 + 10^2 = z^2$ → When $x = 40$ & $y = 30$,

$2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right) + 0 = 2z\frac{dz}{dt}$ $\quad (70)^2 + 10^2 = z^2$

$z \approx 70.7$ ft

$$\frac{dz}{dt} = \frac{(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right)}{z}$$

$$\approx \frac{(40+30)(4+3)}{70.7}$$

$$\approx \boxed{6.93 \text{ ft/s}}$$