PRS Answers
November 16, 2004

1. *Answer: (a).* This problem emphasizes one important application of the EVT which is at the same time a very geometric result; that we can put a continuous function on a closed interval inside a box! On a closed interval, $m \leq f(x) \leq M$, so if we take $A = \max\{|m|, |M|\}$, then on this closed interval, f fits in this sort of box. This idea will ultimately show up in finding bounds for integrals.

2. *Answer: (d).* First answer this question in the easiest form, discussing instantaneous and average velocity - then go back and try to answer again in a way that allows the application of the mean value theorem easily. The fact that (c) is also true, can be shown by using a graph. This is probably the hardest part of the problem.

3. *Answer: False.* This emphasizes the differentiability hypotheses when using the MVT (Note that $f(x)$ is not differentiable at 0).

4. *Answer: (c).* $\dfrac{A_2 - A_1}{r_2 - r_1} = \dfrac{\pi(r_2^2 - r_1^2)}{r_2 - r_1} = \pi(r_2 + r_1)$. On the other hand, by MVT, $\dfrac{A_2 - A_1}{r_2 - r_1} = 2\pi r$ for some $r \in (r_1, r_2)$. Thus, $A_2 - A_1 = 2\pi r(r_2 - r_1)$. This problem is a fairly simple application of the MVT, but the fact that you can get the answer to be exactly the area of that rectangle might surprise you. This problem will be a good reference later on, when we are trying to prove the evaluation part of the Fundamental Theorem of Calculus.