Math 1a Fall 2004
Final Exam Review

1. Ned operates a tour service that offers the following rates:

   - \$200 per person if 50 people (the minimum number to book the tour) go on the tour
   - For each additional person, up to a maximum of 80 people total, the rate per person is reduced by \$2.

   It costs \$6000 (a fixed cost) plus \$32 per person to conduct the tour. How many people does it take to maximize Ned's profit?

2. Use Newton's method to find the positive fourth root of 2. Start with $x_1 = 1$ and find x_3.

3. A balloon ascending at the rate of 12 feet per second is at a height of 80 feet above the ground when a package is dropped from the balloon. How long does it take the package to reach the ground?

4. Consider the definite integral $\int_{-\pi}^{\pi} (\sin x + 1)\, dx$.

   (a) Estimate this integral using four approximating rectangles and right endpoints.
   (b) Use the Midpoint Rule with $n = 5$ to estimate this integral.
   (c) Set up an expression for this integral as a limit of Riemann sums.
   (d) Find the value of this integral exactly.

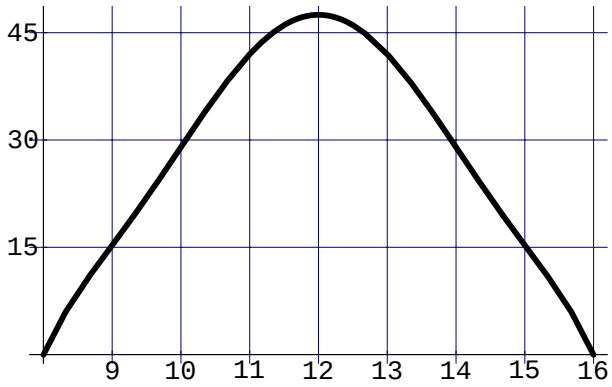
5. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x$, $-1 \le x \le 2$.

6. Find $\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4 + 1}\, dt$.

7. Evaluate each of the following integrals.

   (a) $\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2 + 1}}\, dx$
   (b) $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{4x}{\sqrt{x^2 + 1}}\, dx$
   (c) $\int_1^4 \frac{1}{2\sqrt{y}(1 + \sqrt{y})^2}\, dy$
   (d) $\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta \cos^3\left(\sqrt{\theta}\right)}}\, d\theta$

8. During flu season a health clinic has set up a flu shot program for its patients. The clinic is open Saturday from 8 AM to 4 PM giving flu shots on a first-come, first-serve basis. The clinic has the capacity to serve 30 patients per hour. The function $r(t)$, whose graph is given below, gives the rate at which people are arriving at the clinic for shots.



Express the following quantities in terms of definite integrals. For each quantity, estimate its value using the graph above.

(a) The length of the line at 11 AM

(b) The length of the line at its longest

(c) The number of people who came to the clinic for flu shots

(d) The number of people actually served by the clinic