

Problem Sheet B

1. Consider the bucket from problem 1 in Problem Set 0. Let $h = f(V)$ be the function that gives height as a function of volume. Let V_f be the volume of the full bucket.

$$\text{Let } V_{.25} = \frac{V_f}{4}, \quad V_{.5} = \frac{V_f}{2} \text{ and } V_{.75} = \frac{3V_f}{4}.$$

Put $V_{.25}$, $V_{.5}$ and $V_{.75}$ in ascending order (from small to large).

2. Let $f(x) = |2(x + 3)| + 1$.
- (a) Graph $f(x)$.
 - (b) At which point is f not differentiable?
 - (c) Use your answer to (a) to find $f'(x)$.
3. For each of the following, find f' using the limit definition of derivative. For (a) and (b) explain why your answers make sense from a graphical perspective.
- (a) $f(x) = c$ where c is a constant
 - (b) $f(x) = bx$ where b is a constant
 - (c) $f(x) = ax^2$ where a is a constant
 - (d) $f(x) = ax^2 + bx + c$ where a , b , and c are constants
4. Suppose we are given the equation of a parabola $f(x) = ax^2 + bx + c$. Explain why the sign of a determines whether the parabola opens up or down. Do this twice, first without using calculus and then using calculus.
5. (a) Suppose $f(x) = g(x) + C$. How are the graphs of f and g related? Is $f'(x) = g'(x)$? Why or why not? (Give a graphical explanation.) If not, what is the relationship between f' and g' ?
- (b) Suppose $f(x) = g(x + C)$. How are the graphs of f and g related? Is $f'(x) = g'(x)$? Why or why not? (Give a graphical explanation.) If not, what is the relationship between f' and g' ?
6. Let $f(x) = \frac{1}{x}$.
- (a) Graph f and then graph f' given the graph of f .
 - (b) Use the limit definition of derivative to find $f'(x)$.
 - (c) Make sure your answers to (a) and (b) agree.