Problem Sheet A

1. *Graphs of functions* If possible, sketch a graph of a continuous function defined for all real numbers with all three characteristics listed. If it is impossible to do this, say so, and draw the graph of a function with the three characteristics on the domain $[-1, 1]$.

   We will define continuity in the upcoming weeks. For now, we can very informally say that the graph of a continuous function can be drawn without lifting your pen from the paper. Alternatively, again, very loosely, if x_1 and x_2 are very close togethe then $f(x_1)$ is close to $f(x_2)$. The language of calculus will give us the words to make this definition precise.

   (a) $f(x)$ is positive, increasing, and concave up on $(-\infty, \infty)$

   (b) $f(x)$ is positive, increasing, and concave down on $(-\infty, \infty)$

   (c) $f(x)$ is negative, increasing, and concave down on $(-\infty, \infty)$

   (d) $f(x)$ is negative, decreasing, and concave up on $(-\infty, \infty)$

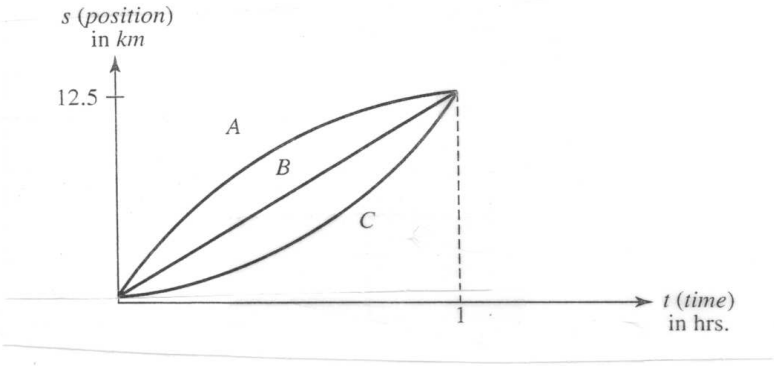
2. *Characterizing Familiar Functions* (If it is not easy for you to come up with functions f and g characterized below, review §1.1-1.6 in your text.)

   (a) Give an example of a function f on the domain $(-\infty, \infty)$ such that $f(x)$ is greater than 1 everywhere and increasing everywhere.

   (b) Give an example of a function g on the domain $(-\infty, \infty)$ such that $g(x)$ changes from increasing to decreasing infinitely many times.

3. *Runners*

   (a) Early one morning three runners leave Washington State University in Pullman and run the Palouse Path[1] joining their town to the University of Idaho. Alicia, Bertha, and Catrina run for 1 hour. The graphs below give position, $s(t)$, as a function of time, for each of the runners. The position function gives position measured from the start of the path from time $t = 0$ to t. The runners' position graphs are labeled A, B, and C, respectively.



      i. Narrate the run, comparing and contrasting the running styles of the three women.

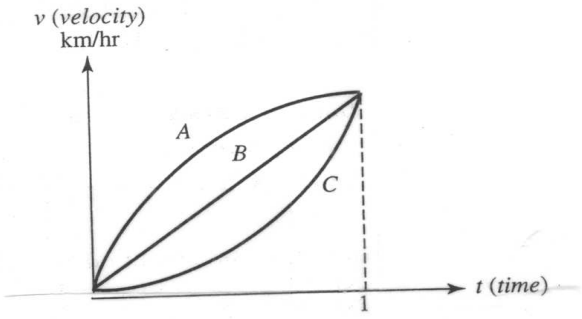
      ii. Compare the average velocities of the three runners.

      iii. Who is ahead after 1 hour of running?

   (b) At lunchtime Amir, Baboucar, and Carlos leave Washington State University and head for the University of Idaho along the same trail. The graphs below give velocity, $v(t)$, as a function of time for each of

[1] This path, officially called the Bill Chipman Palouse Path, is a rails-to-trails conversion completed in the late 1990s. It joins the small college towns of Pullman, Washington, and Moscow, Idaho, along a defunct railway track.

the runners. Each man runs for an hour. Their position graphs are labeled A, B, and C, respectively.



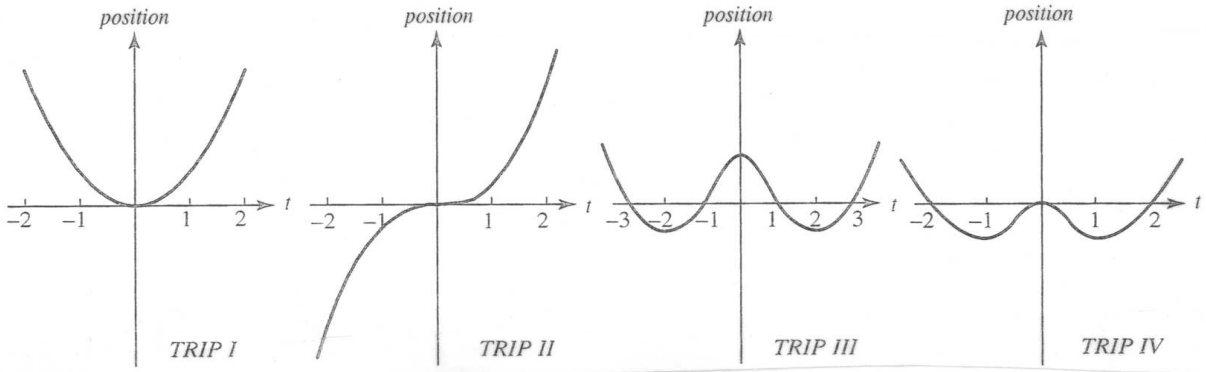
i. Narrate the run. Be sure to mention who is the frontrunner and who is behind after half an hour and then after 1 hour. Compare and contrast the runners' running styles.

ii. Compare the average velocities of the runners for the 1-hour time block.

iii. Compare the average accelerations of the runners for the 1-hour time block.
Note: average acceleration $= \frac{\text{change in velocity}}{\text{change in time}}$.

4. Below are graphs giving information about trips. The trips are all taken along Interstate Route 90, a road that runs east-west through Sturbridge, Massachusetts. The top four graphs give position versus time while the bottom four give velocity versus time.

We use as our benchmark location Sturbridge itself. We will indicate that we are 10 miles east of Sturbridge by writing position $= 10$; we will indicate that we are 10 miles west of Sturbridge by writing position $= -10$. Positive velocity will indicate that we are traveling from west to east; negative velocity will indicate that we are traveling from east to west. We use noon as our benchmark time; noon corresponds to time $t = 0$. Therefore time $t = -2$ is 10:00 a.m.

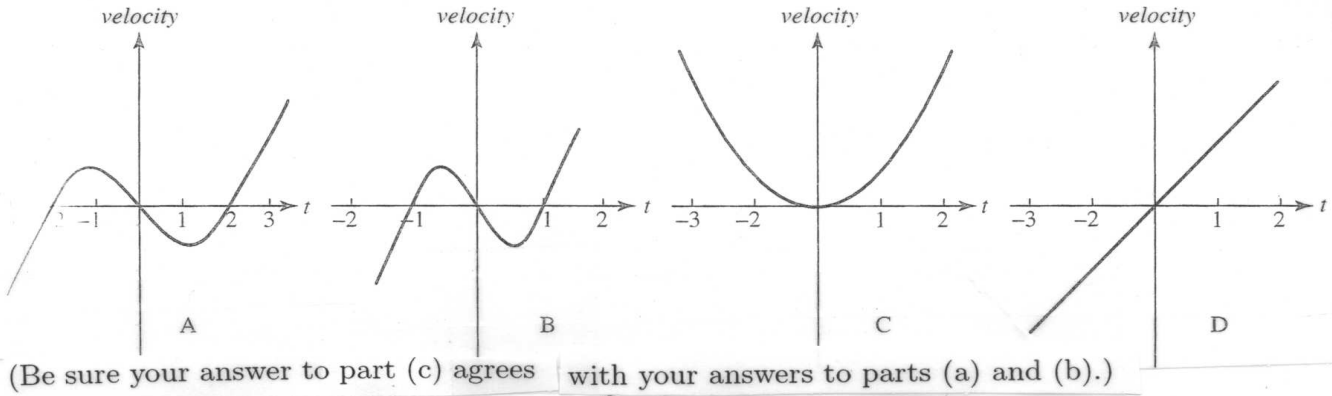


Answer parts (a), (b), and (c) for each of the trips corresponding to graphs I, II, III, and IV.

(a) For what values of t is velocity positive? When is travel from west to east?

(b) For what values of t is velocity negative? When is travel from east to west?

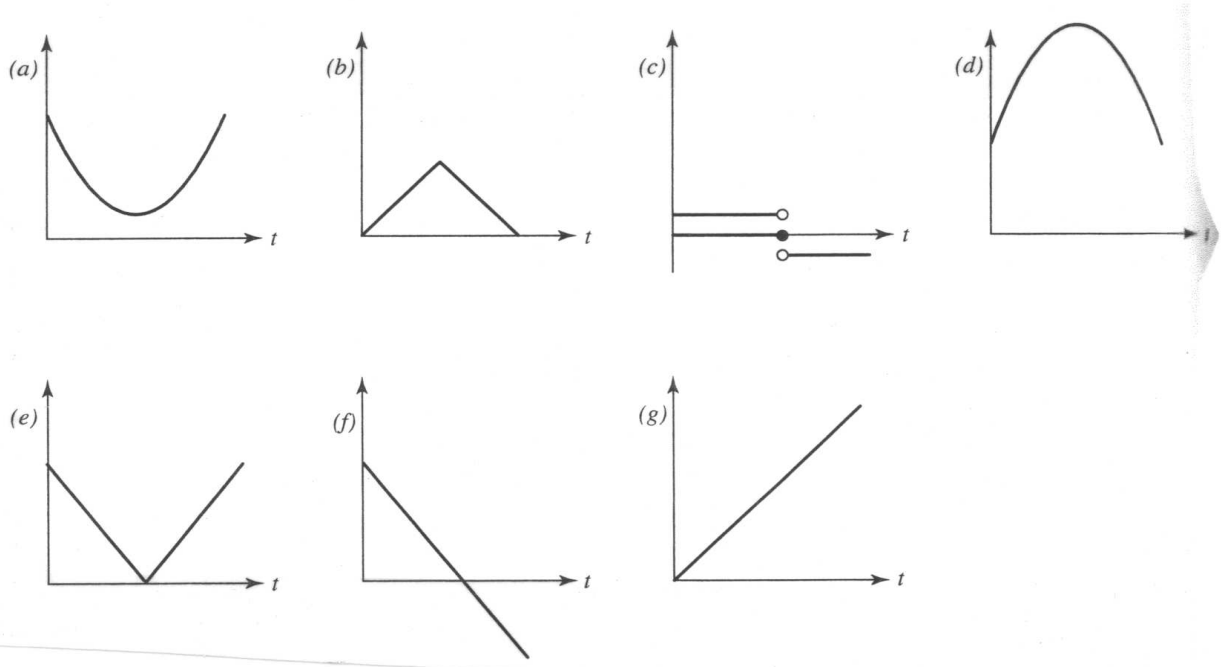
(c) To which trip do each of the following velocity graphs correspond? (Be sure your answer to part (c) agrees with your answers to parts (a) and (b).)



5. Look back at the figures for the previous problem. What characteristic of the graph of position versus time determines the sign of the velocity?

6. An ape with budding consciousness throws a bone straight up into the air from a height of 2 feet. From the seven graphs that follow pick out the one that could be the bone's

   (a) height versus time, (b) velocity versus time, (c) speed versus time.



7. The displacement of an object is given by $d(t) = 2t^5 - 6t^3 + 2t^2 + 1$ miles over the time interval $-2 \leq t \leq 2$ where t is measured in hours. The following questions are designed to be answered with the help of a graphing calculator.

   (a) Approximately when does the object change direction? Please give answers accurate to within 0.1. When you zoom in on the graph here, what do you observe?

   (b) Approximately when is the object's velocity positive? Negative?

   (c) Approximate the object's velocity at time $t = 0$.

8. Consider the following functions:

   $f_1(h) = -2 - h$ with domain $(-\infty, \infty)$

   $f_2(h) = -2 - h$ with domain $(-\infty, 0) \cup (0, \infty)$, i.e. all x except $x = 0$

   $f_3(h) = \dfrac{(-2-h)h}{h}$ with its largest natural domain

   $f_4(h) = \dfrac{(-2-h)h(h-1)}{h(h-1)}$

   $f_5(h) = -2 - h$ for $h \neq 0$ and $f_5(0) = 5$

   Answer the following:

   (a) Two of the functions are identical. Which two are they?

   (b) Sketch $f_k(h)$ for $k = 1, 2, 3, 4, 5$

   (c) For which values of k is $\lim_{h \to 0} f_k(h) = -2$?

   (d) Suppose $g(h) = -2 - h$ for $h \neq 0$ and $g(0) = C$ for some constant C. Is there any value that can be assigned to C so that $\lim_{h \to 0} g(h) \neq -2$?