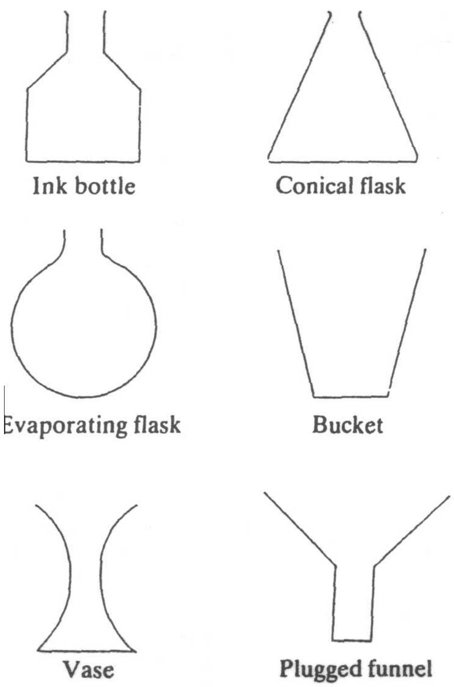
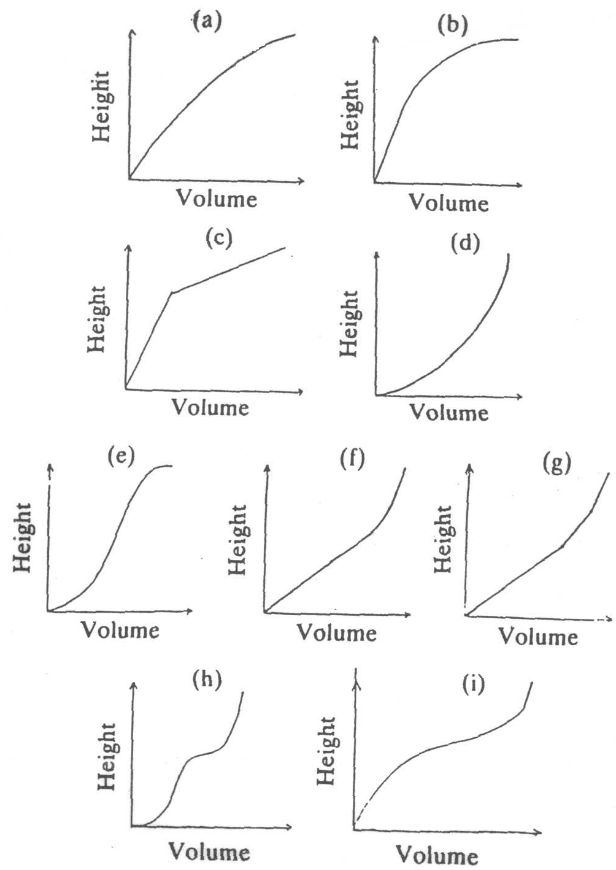
Introductory Assignment

Due on the first day of class, Monday, September 22nd or Tuesday, September 23rd.

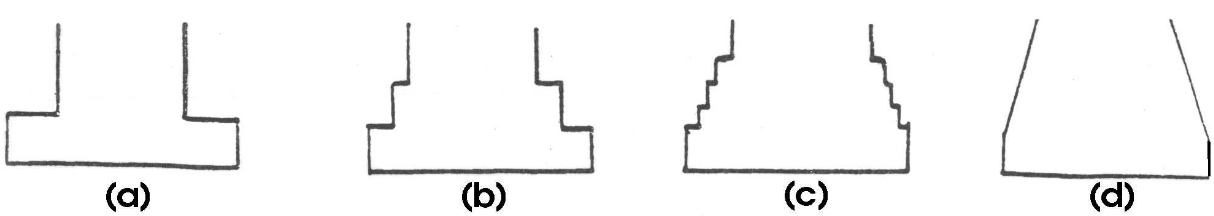
1. *Bottle Calibration Problem*

   Below are 6 bottles and 9 graphs of height versus volume. Choose the correct graph for each bottle. Explain your reasoning clearly and in plain English.





2. Sketch height versus volume graphs for the following sequece of flasks.



3. *Transformations of functions and thier graphs:*

Throughout the course (in class, on homework, and on examinations) we will expect you to be able to sketch the graphs of some basic functions without the aid of a graphing calculator. These functions include

$$y = x^2,\ y = x^3, y = \frac{1}{x},\ y = \frac{1}{x^2},\ y = |x|,$$
$$y = 3^x,\ \text{(and similarly, } y = e^x \text{ given that } e \approx 2.72),\ y = \ln x,$$
$$y = \sin x,\ y = \cos x, \text{and } y = \tan x.$$

Not only should you be able to graph these functions, but you should be able to shift them horizonatally and vertically, stretch or compress them horizontally and vertically, and flip them over the coordinate axes. (For instance, once you can graph $y = 2^x$, then you can graph $y = -2^x + 3$ and $y = 2^{-x}$. Once you can graph $y = |x|$ you can graph $y = |x + 3| - 2$. For a general review of functions, read your text, sections 1.1-1.6. For shifting, flipping, stretching and shrinking, read §1.3.

Do §1.3 #1 and 3 in your textbook. (You can check your answers in the back of the book. If you have trouble, read §1.3 and then try # 5. Only 1 and 3 need to be turned in.)

Much of calculus can be presented in terms of the relationship between position and velocity. Thinking about information that can be gleaned from odometer readings and information that can be gleaned from speedometer readings can be helpful in sorting out ideas in caclulus. (Speed is the absolute value of velocity.) The next three problems deal with velocity and position. The first asks you to use and interpret functional notation and the next two ask you to interpret graphs. All are good warm-ups for the course.

4. Some friends are taking a long car trip. They are traveling east on Route 66 from Flagstaff, Arizona, through New Mexico and Texas and into Oklahoma.

   - Let f be the function that gives the number of miles traveled t hours into the trip, where $t = 0$ denotes the beginning of the trip. For instance, $f(7)$ is the mileage 7 hours into the trip. If the travelers set an odometer to zero at the start of the trip, the output of f would be the reading on the odometer.
   - Let g be the function that gives the car's speed t hours into the trip, where $t = 0$ denotes the beginning of the trip. For instance, $g(7)$ is the car's speed 7 hours into the trip. The output of g corresponds to the speedometer reading.

   Suppose they pass a sign that reads "entering Gallup, New Mexico," h hours into the trip.

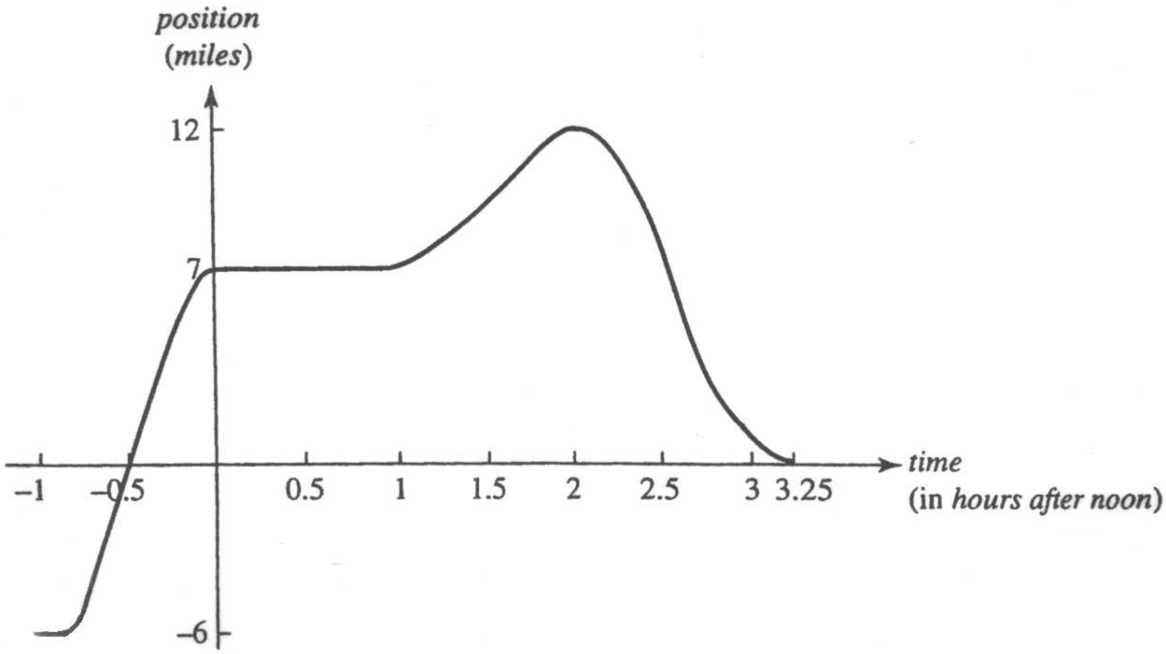
   (a) Write the following expressions using functional notation wherever appropriate.

      i. The car's speed 1 hour before reaching Gallup
      ii. 10 miles per hour slower than the speed of the car entering Gallup
      iii. Half the time it took to reach Gallup
      iv. Their speed 6 hours after reaching Gallup
      v. The distance traveled in the first 2 hours of the trip
      vi. The distance traveled in the second 2 hours of the trip
      vii. The average speed in the first 5 hours of travel (Average speed is computed by dividing the distance traveled by the time elapsed.)
      viii. The average speed between hour 6 of the trip and hour 12 of the trip

   (b) Interpret the following in words.

      i. $f(h + 2)$
      ii. $\frac{1}{2}f(h)$
      iii. $f(\frac{h}{2})$
      iv. $f(h - 2)$
      v. $f(h) - 2$
      vi. $g(h + 2)$
      vii. $g(h) + 2$

5. The graph below gives information about a bike ride along Route 1A, a road going north-south along the coast of Massachusetts from the Boston area, through a town called Lynn, and past Salem. The graph tells us the cyclist's position, p, along the road (in miles) as a function of time: $p = f(t)$. We will use the town of Lynn as our benchmark for measuring position; we indicate that the cyclist is 2 miles north of Lynn by setting position equal to 2 and indicate that the cyclist is 2 miles south of Lynn by writing position equals -2. We let the benchmark time of 12 noon correspond to $t = 0$.



Use the graph to answer the following questions.

(a) Where (in relation to Lynn) and at what time does the bike trip start?

(b) Interpret the intercepts on the horizontal axis and the vertical axis in terms of the trip.

(c) When is the cyclist traveling north? South?

(d) Salem is about 7 miles north of Lynn. Does the cyclist ever reach Salem? If so, approximate the time she gets there. If she comes back through Salem, approximate that time.

(e) What is the cyclist doing between noon and 1:00 p.m. ?

(f) We define **speed** to be the absolute value of velocity. Speed is always nonnegative; it gives no information about direction.

What is the cyclist's average speed between 2:00 p.m. and 3:15 p.m. ?

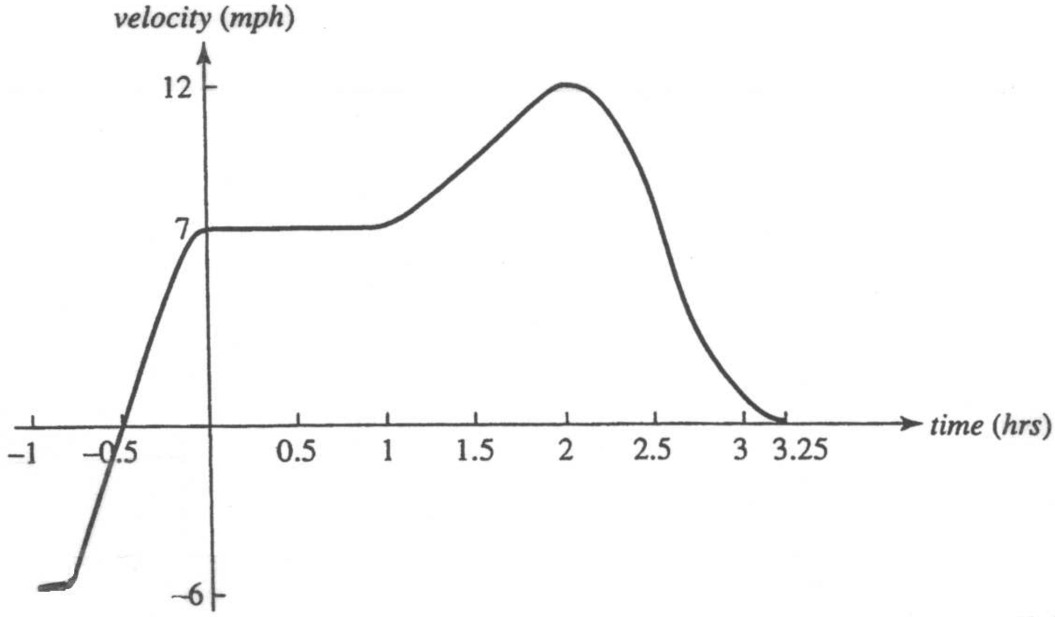
What is the cyclist's average velocity between 2:00 p.m. and 3:15 p.m. ?

(g) Does the cyclist ever turn around and change direction? When?

(h) What is the farthest north of Lynn the cyclist ever gets? What is the farthest south?

6. The graph below is the graph of a different bike ride; now the graph represents velocity, v, as a function of time. $v = f(t)$. When velocity is negative the rider is moving south, and when velocity is positive she is moving north. Here again, time $t = 0$ represents noon.

While this graph is similar in *shape* to the previous graph, the two graphs represent completely different bike rides.



Use the graph to answer the following questions.

(a) During what time period(s) is the cyclist riding south? North?

(b) Interpret the intercepts on the horizontal axis and the intercept on the vertical axis in terms of the trip.

(c) When is the cyclist traveling most rapidly south? Most rapidly north?

(d) Does the cyclist ever change direction? If so, when?

(e) Does the cyclist ever stop? If so, when?

(f) Can you determine where the cyclist started her trip?

(g) What is the cyclist's maximum speed? Minimum speed?