Name: _____

Mathematics 1a
Final Examination
May 24, 1995

Please circle the name of your section leader:

Srdjan Divac Robert Kaplan (10:00) Robert Kaplan (11:00) Esther Silberstein

Show all your work.

Question	Points	Score
1	6	
2	6	
3	8	
4	6	
5	7	
6	8	
7	9	
8	9	
9	10	
10	10	
11	8	
12	13	
Total	100	

1. (6 points) Use the **definition of derivative** to find $f'(x)$ if $f(x) = \dfrac{1}{1-x}$.

2. (6 points) Find the derivatives of the following functions:

(a) $g(x) = x^3 \cdot e^x$

(b) $k(x) = \ln(\sin x + \cos x)$

(c) $f(x) = \dfrac{\arcsin(\sqrt{x})}{1+x}$

3. (8 points) Find antiderivatives of the following functions:

(a) $f(t) = 2t^7 - 13t + \dfrac{3}{\cos^2 t}$

(b) $h(t) = \dfrac{(\ln 3t)^3}{t}$

(c) $g(t) = \dfrac{1 - 3t + t^2}{t^2}$

(d) $m(t) = \dfrac{e^t}{\sqrt{1 - e^{2t}}}$

4. (6 points) Given the curve $y \tan x = \cos y$,

(a) find $\frac{dy}{dx}$

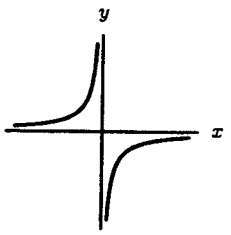
(b) Find the equation of the tangent to the curve at $\left(0, \frac{\pi}{2}\right)$.

5. (7 points) Find the area between $y = \cos x$ and $y = x^2 - \frac{\pi^2}{4}$.

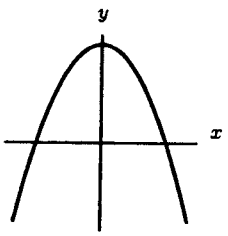
6. (8 points) Each graph in the right-hand column represents the **second derivative** of some function in the left-hand column. Match the functions and their second derivatives.

Functions

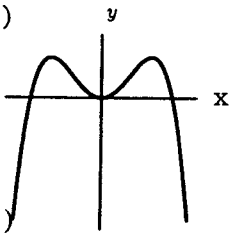
(a)



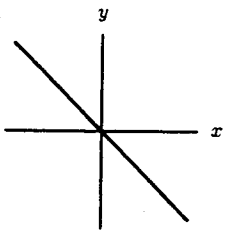
(b)



(c)

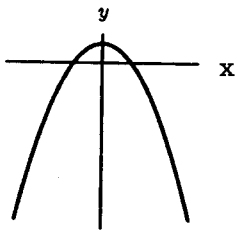


(d)

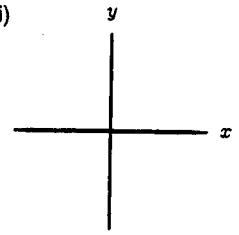


Second Derivatives

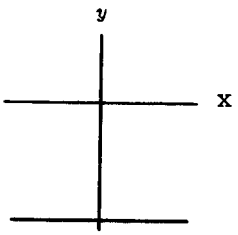
(i)



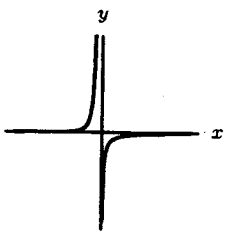
(ii)



(iii)



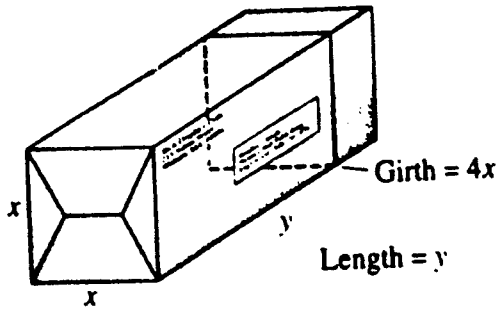
(iv)



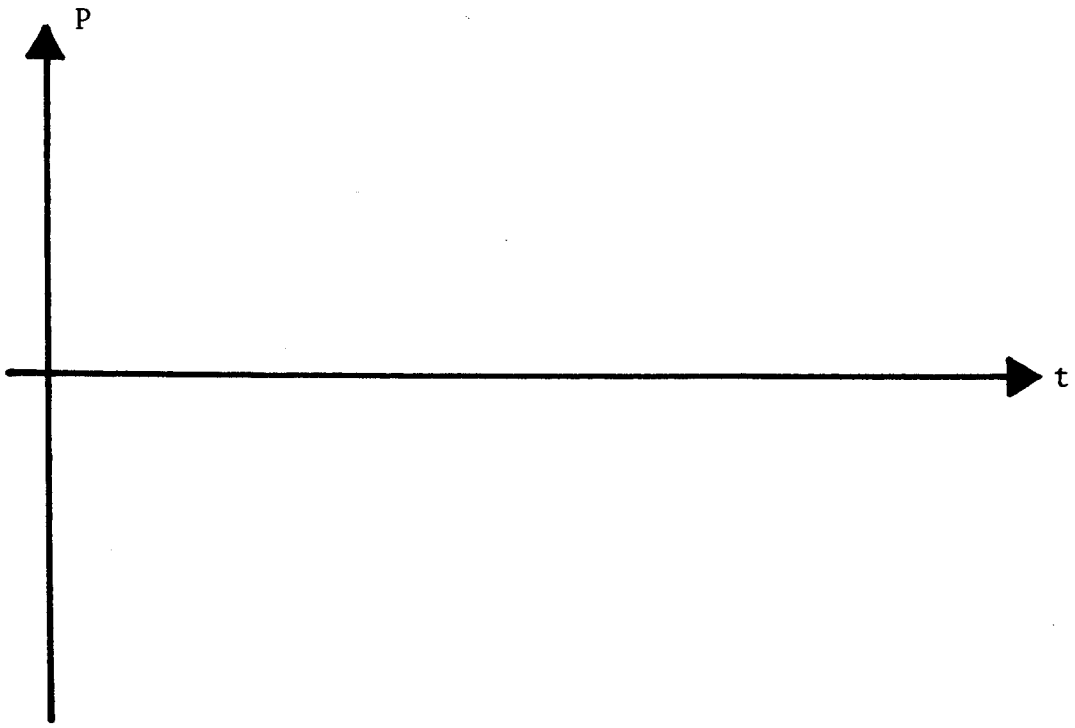
Function (a) has second derivative _____.
Function (b) has second derivative _____
Function (c) has second derivative _____.
Function (d) has second derivative _____.

7. (9 points) A searchlight in a lighthouse 15 miles off a straight shore is revolving at the rate of 2 revolutions/minute. At what speed does the beam of light pass a point 20 miles down the shore from the lighthouse?

8. (9 points) The U. S. Postal Service will accept a box for domestic equipment only if the sum of its length and girth (distance around) does not exceed 108 inches. Find the dimensions of the largest acceptable box with a square end. (Greatest volume!)



9. (10 points) (a) Sketch a representative family of solution curves for the differential equation $\frac{dP}{dt} = \cos P$ for $-2\pi \leq 0 \leq 2\pi$ on the axis below.



(b) What happens to $P(t)$ as $t \to \infty$ if $P(0) = 6$?

(c) What happens to $P(t)$ as $t \to \infty$ if $P(0) = \frac{3\pi}{2}$?

10. (10 points) Eugene Saperstein is driving south on the Panamaniac Highway in his "Volga" sedan. Suddenly, having seen a roadblock, he steps on the brakes and decelerates at $-20m/s^2$. If it takes him 90m to come to a stop, how fast was he driving (in m/s) when he stepped on the brakes?

11. (8 points) (a) How many subdivisions would be needed to approximate $\int_0^1 \frac{(1+x^2)^5}{100}\,dx$ with an error of no more than 0.11?

(b) Using the number of subdivisions n you found in part (a), find an upper bound U for $\int_0^1 \frac{(1+x^2)}{100}dx$.

(c) Using the number of subdivisions n you found in part (a), find a lower bound L for $\int_0^1 \frac{(1+x^2)^5}{100}\,dx$

12. (13 points) Consider the function $f(x) = \dfrac{\ln x}{x}$.

(a) Show that $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(b) Show that $f''(x) = \dfrac{2\ln x - 3}{x^3}$.

(c) In what intervals is f increasing?

(d) In what intervals is f concave up?

(e) Provide the coordinates of all local max and local min, if any, and indicate what they are. Do not approximate.

Continued on next page.

12, continued.

(f) Provide the coordinate of all inflection points. Do not approximate.

(g) Does this function have any vertical asymptotes? Explain.

(h) Does this function have any horizontal asymptotes?

(i) Sketch the graph of the above function, labeling (without approximating) all intercepts, stationary points, inflection points, and asymptotes. Choose a scale such that all features appear clear.