

MATH 2a FINAL REVIEW PROBLEMS - SOLUTIONS

FALL 1999

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① $f'(x) = (2^{2+6x^3}) (\ln 2) (18x^2)$

② $f'(\theta) = (\cos 5\theta)(-\sin 5\theta)(5)$
 $= -30 \cos 5\theta \sin 5\theta$

③ $f'(x) = \frac{3}{\sqrt{1-(3x+1)^2}}$

④ $f'(x) = \frac{\frac{1}{x}}{1+\ln x} = \frac{1}{x(1+\ln x)}$

⑤ write $y = (\ln x)^{\tan^{-1} x}$ and use logarithmic differentiation

$\ln y = \tan^{-1} x \cdot \ln(\ln x)$

$\frac{1}{y} \frac{dy}{dx} = \tan^{-1} x \cdot \frac{1}{x \ln x} + \frac{1}{1+x^2} \cdot \ln(\ln x)$

$\frac{1}{y} \frac{dy}{dx} = \frac{\tan^{-1} x}{x \ln x} + \frac{\ln(\ln x)}{1+x^2}$

$\frac{dy}{dx} = f'(x) = \left[\frac{\tan^{-1} x}{x \ln x} + \frac{\ln(\ln x)}{1+x^2} \right] (\ln x)^{\tan^{-1} x}$

⑥ $f'(x) = \frac{d}{dx} \int_2^{e^x} \frac{\ln z}{z} dz$

$= \frac{\ln(e^x)}{e^x} \cdot \frac{d}{dx}(e^x) - \frac{\ln 2}{2} \cdot \frac{d}{dx}(2)$
 (by Leibnitz' Rule)

$= \frac{x}{e^x} \cdot e^x - 0 = \boxed{x}$

③ $f(x) = \int_0^x (t^2 - 3t + 2) dt$
 $= \left[\frac{t^3}{3} - \frac{3t^2}{2} + 2t \right]_{t=0}^{t=x}$

$f(x) = \frac{1}{3} x^3 - \frac{3}{2} x^2 + 2x$

$f'(x) = x^2 - 3x + 2$

$f''(x) = 2x - 3$

X-Intercepts: $x \left(\frac{1}{3} x^2 - \frac{3}{2} x + 2 \right) = 0$

$\frac{1}{6} x (2x^2 - 9x + 12) = 0$

$x=0 \quad x = \frac{9 \pm \sqrt{81-96}}{4}$

$\Rightarrow (0, 0)$ only.

CRIT PTS: $(x-2)(x-1) = 0$

$\boxed{x=1} \quad \boxed{x=2}$

$f'(1) = -1 \Rightarrow \text{Rel. MAX AT } x=1$

$f'(2) = +1 \Rightarrow \text{Rel. MIN AT } x=2$

$f(1) = \frac{1}{3} - \frac{3}{2} + 2 = \frac{5}{6}$

$f(2) = \frac{8}{3} - 6 + 4 = \frac{2}{3}$

P.O.I: $2x - 3 = 0 \quad \boxed{x = \frac{3}{2}}$

$f\left(\frac{3}{2}\right) = \frac{1}{3} \left(\frac{27}{8}\right) - \frac{27}{8} + 3$

$= \frac{27}{24} - \frac{54}{24} + \frac{72}{24} = \frac{18}{24} = \frac{3}{4}$

② a) $\frac{3x^2}{2} + \frac{x^{e+1}}{e+1} + e^x + C$

b) $\int \frac{e^x dx}{1+(e^x)^2} \quad u=e^x \quad du=e^x dx$

$= \int \frac{du}{1+u^2} = \tan^{-1} u + C$

$= \tan^{-1}(e^x) + C$

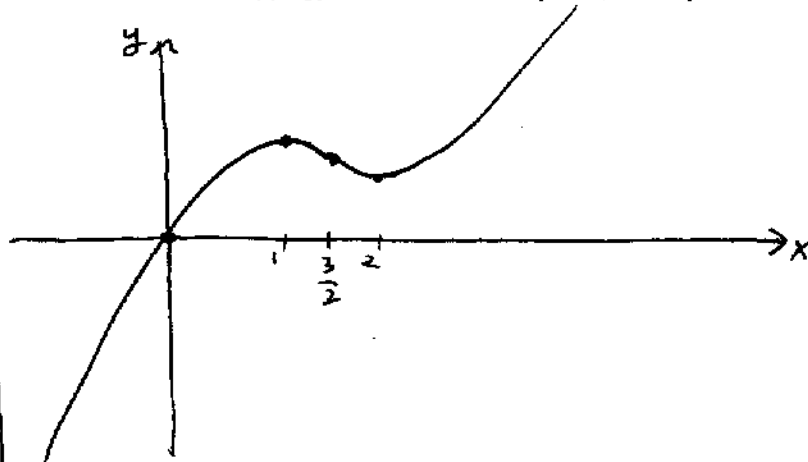
c) $\int_1^e \frac{e(\ln x)^2}{x} dx \quad u=\ln x \quad du = \frac{dx}{x}$

$= \int_0^1 u^2 du = \frac{u^3}{3} \Big|_0^1 = \frac{1}{3}$

d) $\int \frac{dx}{(x+1)^2 + 2} = \frac{1}{2} \int \frac{dx}{\left(\frac{x+1}{\sqrt{2}}\right)^2 + 1}$

$u = \frac{x+1}{\sqrt{2}} \quad du = \frac{dx}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} \int \frac{du}{u^2+1} = \frac{1}{\sqrt{2}} \tan^{-1} u = \boxed{\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x+1}{\sqrt{2}}\right) + C}$

THERE ARE NO ASYMPTOTES.



④ $f(x) = x^2 e^{-x} \quad x \geq 0$
 $f'(x) = -x^2 e^{-x} + 2x e^{-x} = (2x - x^2) e^{-x}$
 $f''(x) = -(2x - x^2) e^{-x} + (2 - 2x) e^{-x}$
 $= (x^2 - 4x + 2) e^{-x}$

$f(0) = 0 \Rightarrow (0, 0)$ on graph.
 Since $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = 0$
 Graph continuous at $(0, 0)$.

no other x -intercepts
 $\lim_{x \rightarrow +\infty} (x^2 e^{-x}) = \lim_{x \rightarrow +\infty} \frac{x^2}{e^x} \rightarrow \frac{\infty}{\infty}$
 $\therefore = \lim_{x \rightarrow +\infty} \left(\frac{2x}{e^x} \right) = \lim_{x \rightarrow +\infty} \left(\frac{2}{e^x} \right) = 0$

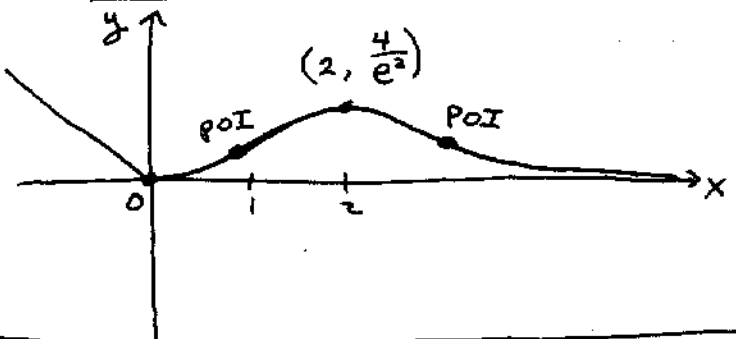
So $\lim_{x \rightarrow +\infty} f(x) = 0$ (Horiz Asymptote)

CRIT PTS: $2x - x^2 = x(2-x) = 0$
 $\Rightarrow x=0 \quad x=2$
 $f''(2) = -2e^{-2} < 0 \therefore$ (Rel. MAX)

POI: $x^2 - 4x + 2 = 0$
 $x = \frac{4 \pm \sqrt{16-8}}{2}$
 $x = 2 \pm \sqrt{2}$

$\lim_{x \rightarrow 0^-} f'(x) = -1 \quad \lim_{x \rightarrow 0^+} f'(x) = 0$

slopes don't MATCH, so graph is not differentiable at 0



⑤ Let $x = \#$ of add'l trees to be planted.
 TOTAL TREES = $30 + x$
 # apples per tree = $400 - 7x$
 TOTAL APPLES = $f(x) = (30+x)(400-7x)$
 $= 12000 + 190x - 7x^2$

$f'(x) = 190 - 14x = 0$
 $x = \frac{190}{14} = 13 \frac{4}{7}$

$f(13) = (43)(309) = 13287$
 $f(14) = (44)(302) = 13288$
 She should plant 14 additional trees for a total of 44 trees.

⑥

$a = \frac{dv}{dt} = -20 \text{ ft/sec}^2$

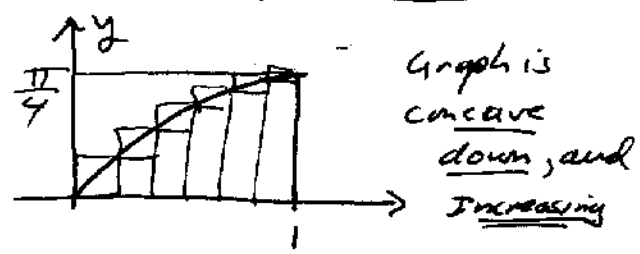
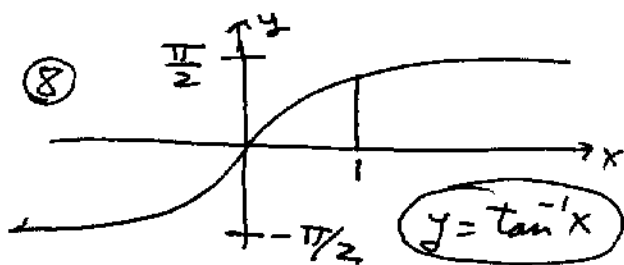
Let $v(0) = v_0$
 $v(t) = -20t + C$
 $v(0) = C = v_0 \Rightarrow v(t) = v_0 - 20t$
 So $\frac{dx}{dt} = v_0 - 20t$
 $x(t) = v_0 t - 10t^2 + C'$

we can take $x(0) = 0$, and if we let $\pi =$ duration of skid we'll have $x(\pi) = 250$.

$x(0) = C' = 0 \Rightarrow x(t) = v_0 t - 10t^2$
 we also know that $v(\pi) = 0$
 So $\begin{cases} v_0 - 20\pi = 0 \\ v_0 \pi - 10\pi^2 = 250 \end{cases}$
 $v_0 = 20\pi \Rightarrow 20\pi^2 - 10\pi^2 = 250$
 $10\pi^2 = 250 \quad \pi^2 = 25$

So $\pi = 5 \text{ sec}$ and $v_0 = 100 \text{ feet/sec}$

⑦ $\cos(xy) + e^{2y} = e^y \quad \left(\frac{\pi}{4}, 2 \right)$
 $-\sin(xy) \left[x \cdot \frac{dy}{dx} + y \right] + e^{2y} \cdot 2 \frac{dy}{dx} = 0$
 $\frac{dy}{dx} [2e^{2y} - x \sin xy] = y \sin(xy)$
 $\frac{dy}{dx} = \frac{y \sin xy}{2e^{2y} - x \sin xy}$
 $\frac{dy}{dx} \Big|_{\left(\frac{\pi}{4}, 2 \right)} = \frac{2 \cdot 1}{2e^4 - \frac{\pi}{4}} = \frac{8}{8e^4 - \pi}$
 TANGENT LINE IS:
 $y - 2 = \left(\frac{8}{8e^4 - \pi} \right) \left(x - \frac{\pi}{4} \right)$



- a) $L(50)$ underestimates \Rightarrow **TRUE**
- b) $R(50)$ over, $T(50)$ under \Rightarrow **FALSE**
- c) $T(50)$ under \Rightarrow **FALSE**
- d) $L(50), L(100)$ both under, $L(100)$ closer \Rightarrow **TRUE**
- e) $f(1) = \pi/4, \int_0^1 \tan^{-1}(x) dx < \frac{\pi}{4} \cdot 1$ **FALSE**

9) $P(0) = 200, P(t) = Ae^{kt}, P(0) = A = 200$
 So $P(t) = 200e^{kt}$
 $P(2) = 200e^{2k} = 800 \Rightarrow e^{2k} = 4$

a) So $2k = \ln 4 = 2 \ln 2$
 $\Rightarrow k = \ln 2 \Rightarrow P(t) = 200(e^{\ln 2})^t$
 or $P(t) = 200 \cdot 2^t$
 $P(5) = 200 \cdot 32 = 6400$ toads.

b) $P = 200 \cdot 2^t$ Area $A = \pi r^2$
 Initial density $\frac{200 \text{ TOADS}}{\pi \text{ mi}^2}$

After 5 years, Density = $\frac{6400 \text{ TOADS}}{\pi r^2} = \frac{200 \text{ TOADS}}{\pi \text{ mi}^2}$
 $\Rightarrow 200 \pi r^2 = 6400 \pi \text{ mi}^2$
 $\Rightarrow r^2 = 32 \text{ mi}^2 \Rightarrow r = 4\sqrt{2} \text{ mi}$

c) $\frac{dP}{dt} = 200 \cdot 2^t \cdot \ln 2$ $\left. \frac{dP}{dt} \right|_{t=5} = 6400 \ln 2$
 $\frac{P}{A} = \frac{P}{\pi r^2} = \frac{200}{\pi}$ (constant)
 So $P = 200 r^2$
 $\frac{dP}{dt} = 400 r \cdot \frac{dr}{dt}$
 $6400 \ln 2 = 400 \cdot 4\sqrt{2} \cdot \frac{dr}{dt}$
 $\Rightarrow \left. \frac{dr}{dt} \right|_{t=5} = \frac{4 \ln 2}{\sqrt{2}}$

10 a) $f(x) = e^{\tan^{-1}(\ln x)}$
 $f'(x) = e^{\tan^{-1}(\ln x)} \left[\frac{(1/x)}{1+(\ln x)^2} \right]$

b) $x^2 y^3 + 3x e^y - 4 \ln x = 5$
 $x^2 \cdot 3y^2 \frac{dy}{dx} + 2xy^3 + 3x e^y \frac{dy}{dx} + 3e^y \frac{y}{x} - \frac{4}{x} = 0$
 $(3x^2 y^2 + 3x e^y) \frac{dy}{dx} = \frac{4}{x} - 2xy^3 - 3e^y$
 $\frac{dy}{dx} = \frac{4/x - 2xy^3 - 3e^y}{3x^2 y^2 + 3x e^y}$

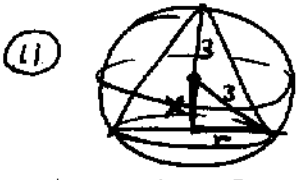
c) $y = \int_0^{\ln(4x)} \sin(e^x) dx = \text{constant}$
 So $\frac{dy}{dx} = 0$.

d) $f(x) = [x^2 e^{3x} + \ln x]^5$
 $f'(x) = 5[x^2 e^{3x} + \ln x]^4 [2x e^{3x} \cdot 3 + \frac{1}{x}]$
 $= 5[x^2 e^{3x} + \ln x]^4 [3x^2 e^{3x} + 2x e^{3x} + \frac{1}{x}]$

e) WRITE $y = x^{\ln x}$
 $\ln y = \ln(x^{\ln x}) = (\ln x)^2$
 $\frac{1}{y} \frac{dy}{dx} = 2(\ln x) \cdot \frac{1}{x}$
 So $\frac{dy}{dx} = \left(\frac{2 \ln x}{x} \right) x^{\ln x}$

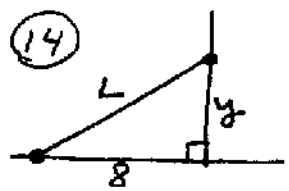
f) $f(x) = 7^{-4x}$
 $f'(x) = (7^{-4x}) (\ln 7) (-4)$

g) $f(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$
 $f'(x) = \frac{(e^{2x} + 1)(2e^{2x}) - (e^{2x} - 1)(2e^{2x})}{(e^{2x} + 1)^2}$
 $= \frac{4e^{2x}}{(e^{2x} + 1)^2}$
 $f'(0) = \frac{4}{2^2} = 1$



(11) height = 3 + x
 radius = r
 $x^2 + r^2 = 3^2 = 9$
 $V = \frac{1}{3} \pi r^2 h$

$V(x) = \frac{1}{3} \pi (9 - x^2)(3 + x)$
 $V(x) = \frac{1}{3} \pi [27 + 9x - 3x^2 - x^3]$
 $\frac{dV}{dx} = \frac{\pi}{3} [9 - 6x - 3x^2] = 0$
 $\pi(3 - 2x - x^2) = 0$
 $x^2 + 2x - 3 = 0$
 $(x + 3)(x - 1) = 0$
 $x = 1$ CRITICAL POINT
 Boundary: $x = -3 \Rightarrow V = 0$
 $x = +3 \Rightarrow V = 0$
 $V(1) = \frac{\pi}{3} (8)(4) = \frac{32\pi}{3}$
 RADIUS = $2\sqrt{2}$ height = 4



(14) $6y + y^2 = L^2$
 $y \frac{dy}{dt} = L \frac{dL}{dt}$
 when $y = 6 \Rightarrow L = 10$ ft.
 If $\frac{dy}{dt} = 4$ ft/sec $\Rightarrow 6 \cdot 4 = 10 \frac{dL}{dt}$
 $\Rightarrow \frac{dL}{dt} = \frac{24}{10} = 2.4$ ft/sec

(15) (a) $\int_{e^2}^{e^3} (\frac{3}{x} + \frac{2x}{e^6 - e^x}) dx = [3 \ln|x| + \frac{x^2}{e^6 - e^x}]_{x=e^2}^{x=e^3}$
 $= (3 \ln(e^3) + \frac{e^6}{e^6 - e^3}) - (3 \ln(e^2) + \frac{e^4}{e^6 - e^2})$
 $= 9 - 6 + \frac{e^6 - e^4}{e^6 - e^2} = 3 + 1 = 4$

(b) $\int \frac{2x+3}{\sqrt{4-x^2}} dx = \int \frac{2x dx}{\sqrt{4-x^2}} + \int \frac{3 dx}{\sqrt{4-x^2}}$
 1st: $u = 4 - x^2 \Rightarrow \frac{-du}{2} = -\frac{1}{2} du = -\frac{2x dx}{2}$
 $= -2 \sqrt{4-x^2}$
 2nd: $= \int \frac{3 dx}{\sqrt{4-x^2}} = 3 \int \frac{du}{\sqrt{4-u^2}} = 3 \sin^{-1} \frac{u}{2}$
 $u = \frac{x}{2} \quad dx = 2 du \quad = 3 \sin^{-1}(\frac{x}{2})$
 So $\int \frac{2x+3}{\sqrt{4-x^2}} dx = -2 \sqrt{4-x^2} + 3 \sin^{-1}(\frac{x}{2}) + C$

(c) $\int_0^{\sqrt{\pi}} x \sin(x^2) dx = \frac{1}{2} \int_0^{\pi} \sin u du = \frac{1}{2} [-\cos u]_0^{\pi}$
 $u = x^2 \quad du = 2x dx \quad = -\frac{1}{2} (-1 - 1) = 1$

(d) $\int e^{-3t} dt = -\frac{1}{3} e^{-3t} + C$
 (e) $\int_1^2 (x + x^{-2}) dx = [\frac{1}{2} x^2 - \frac{1}{x}]_{x=1}^{x=2} = (2 - \frac{1}{2}) - (\frac{1}{2} - 1) = 2$

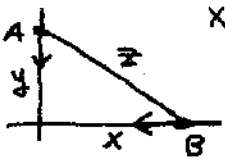
(f) $\int_1^2 \frac{\ln x}{x} dx = \int_0^{\ln 2} u du = \frac{1}{2} u^2 \Big|_0^{\ln 2} = \frac{1}{2} (\ln 2)^2$
 $u = \ln x \quad du = \frac{dx}{x}$

(12) (a) $p^{(20)}(x) = 3 \cos x$
 $p^{(1)}(x) = -3 \sin x$
 (b) $u = 3e^{2x}$
 $u' = 3 \cdot 2^1 e^{2x}$
 $u'' = 3 \cdot 2^2 e^{2x}, \text{ etc}$
 $\dots \Rightarrow u^{(21)} = 3 \cdot 2^{21} e^{2x}$
 (c) $f^{(21)} = 0$ since all terms beyond 12th will vanish.

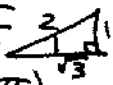
(13) $a(t) = -\frac{1}{(t+1)^2} = \frac{dv}{dt}$
 $v(t) = +\frac{1}{t+1} + C$
 $v(0) = 1 + C = 1 \Rightarrow C = 0$
 So $v(t) = \frac{1}{t+1} = \frac{dy}{dt}$
 $y(t) = \ln(t+1) + C'$
 $y(0) = C' = 1 \Rightarrow y = \ln(t+1) + 1$

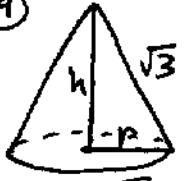
monkey reads $y = 30$
 when $30 = \ln(t+1) + 1$
 $\ln(t+1) = 29$
 $t+1 = e^{29}$
 $t = e^{29} - 1$
 (NOT IN MONKEY'S LIFETIME!)

(g) $u = \sin 3x \quad du = 3 \cos 3x dx$
 $\Rightarrow \int \frac{1}{3} u^2 du = \frac{1}{9} u^3 + C$
 $= \frac{1}{9} \sin^3 3x + C$
 (h) $\int \frac{dx}{1 + (\frac{3x}{2})^2} \quad u = \frac{3x}{2}$
 $dx = \frac{2}{3} du$
 $= \frac{1}{6} \int \frac{du}{1+u^2} = \frac{1}{6} \tan^{-1} u + C = \frac{1}{6} \tan^{-1}(\frac{3x}{2}) + C$

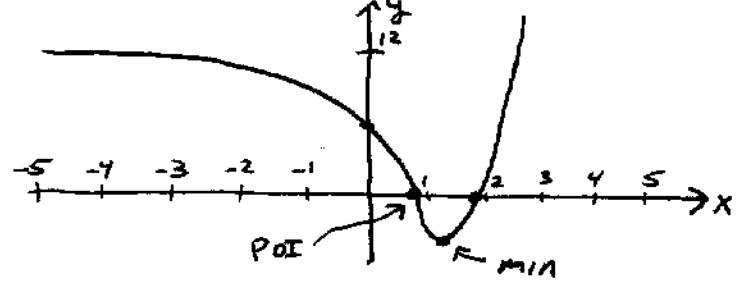
16)  $x^2 + y^2 = z^2$
 $\frac{dx}{dt} = -520 \frac{mi}{hr}$
 $\frac{dy}{dt} = -520 \frac{mi}{hr}$
 $\lambda x \frac{dx}{dt} + \lambda y \frac{dy}{dt} = \lambda z \frac{dz}{dt}$
 when $y=5$ and $x=12$, then $z=13$.
 $\Rightarrow (12)(-520) + (5)(-520) = 13 \frac{dz}{dt}$
 $\Rightarrow \frac{dz}{dt} = -\frac{17}{13}(520) = -680 \frac{mi}{hr}$

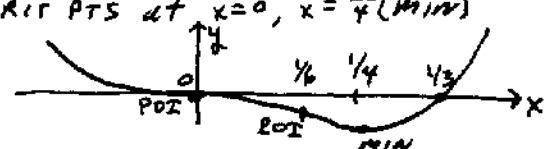
17) $\int_0^x f(t) dt = 3x^2 + e^x - \cos x$
 $\frac{d}{dx} \int_0^x f(t) dt = f(x)$
 So $f(x) = 6x + e^x + \sin x$
 $\therefore f(2) = 12 + e^2 + \sin 2$

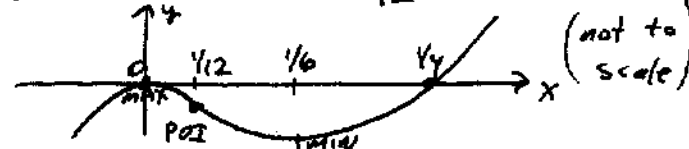
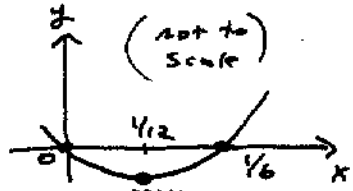
18) $f(x) = \cos x$
 $f'(x) = -\sin x$
 $30^\circ = \frac{\pi}{6}$ 
 $f(x) \approx f(\frac{\pi}{6}) + f'(\frac{\pi}{6})(x - \frac{\pi}{6})$
 $f(\frac{\pi}{6}) = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} \approx 0.866$
 $f'(\frac{\pi}{6}) = -\sin(\frac{\pi}{6}) = -\frac{1}{2} = -0.5$
 $x = 31.8^\circ = \frac{31.8}{180} \pi$ radians
 $\cos(31.8^\circ) \approx 0.866 - \frac{1}{2} (\frac{1.8}{180}) \pi$
 $\approx 0.866 - \frac{3.1416}{200}$
 $\approx 0.866 - 0.015708 \approx 0.850$
 (Actual value = 0.84989...)

19)  $r^2 + h^2 = 3$
 $V = \frac{1}{3} \pi r^2 h$
 $V(h) = \frac{1}{3} \pi (3 - h^2) h$
 $= \frac{1}{3} \pi (3h - h^3)$
 $0 \leq h \leq \sqrt{3}$
 $V'(h) = \frac{1}{3} \pi (3 - 3h^2) = \pi (1 - h^2) = 0$
 $\Rightarrow h=1$ $r=\sqrt{2}$
 $V = \frac{1}{3} \pi \cdot 2 \cdot 1 = \frac{2\pi}{3} = V$
 (endpts give $v=0$)

20) $f(x) = e^{2x} - 8e^x + 12 = (e^x - 6)(e^x - 2)$
 $f'(x) = 2e^{2x} - 8e^x = 2e^x(e^x - 4)$
 $f''(x) = 4e^{2x} - 8e^x = 4e^x(e^x - 2)$
y-intercept: $f(0) = 1 - 8 + 12 = 5$ $(0, 5)$
x-intercepts: $(e^x - 6)(e^x - 2) = 0$
 $e^x = 6$ or $e^x = 2$ $(\ln 2, 0)$
 $x = \ln 6$ or $x = \ln 2$ $(\ln 6, 0)$
CRIT PTS: $2e^x(e^x - 4) = 0$
 $\Rightarrow e^x = 4$ $x = \ln 4$ $(\ln 4, -4)$ REL. MIN
 $f(\ln 4) = (4 - 6)(4 - 2) = -4$
 $f''(\ln 4) = 4 \cdot 4(2) = 32 > 0$
POI: $4e^x(e^x - 2) = 0$ $(\ln 2, 0)$ is a pt. of inflection
 $\Rightarrow e^x = 2$ $x = \ln 2$
Asymptotes: $\lim_{x \rightarrow -\infty} f(x) = (-6)(-2) = 12$



21) $f(x) = x^4 - \frac{1}{3}x^3$
 $f'(x) = 4x^3 - x^2$ $f''(x) = 12x^2 - 2x$
f(x): x-int at $x=0, x=\frac{1}{3}$ POI at $x=0$
CRIT PTS at $x=0, x=\frac{1}{4}$ (MIN) $x=\frac{1}{6}$
 (NOT TO SCALE)

f'(x): x-int. at $x=0, x=\frac{1}{4}$
CRIT PTS AT $x=0, x=\frac{1}{6}$
 $f''(x) = 24x - 2 \Rightarrow$ POI AT $x=\frac{1}{12}$
 (not to scale)
f''(x): x-int. at $x=0, x=\frac{1}{6}$
CRIT PT AT $x=\frac{1}{12}$
 $f'''(x) = 24 \neq 0$ NO POI.
 (not to scale)

(22) $y = \frac{x^2 - x + 1}{x - 1} = x + \frac{1}{x - 1}$

$\frac{dy}{dx} = 1 - \frac{1}{(x-1)^2}$

$\frac{d^2y}{dx^2} = + \frac{2}{(x-1)^3}$

y-intercept at (0, -1)

no x-intercepts.

CRIT PT where $\frac{1}{(x-1)^2} = 1$

$\Rightarrow (x-1)^2 = 1$

$x-1 = \pm 1 \Rightarrow x=0, x=2$

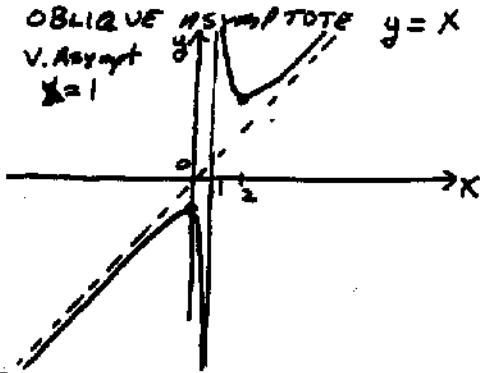
$f''(0) = -2 < 0 \Rightarrow$ LOCAL MAX (0, -1)

$f''(2) = +2 > 0 \Rightarrow$ LOCAL MIN (2, 3)

NO PTS. OF INFLECTION

OBLIQUE ASYMPTOTE $y = x$

V. Asympt $x=1$



(23) $f(x) = \frac{x-1}{x^2} = \frac{1}{x} - \frac{1}{x^2}$

$f'(x) = -\frac{1}{x^2} + \frac{2}{x^3} = \frac{2-x}{x^3}$

$f''(x) = +\frac{2}{x^3} - \frac{6}{x^4} = \frac{2x-6}{x^4}$

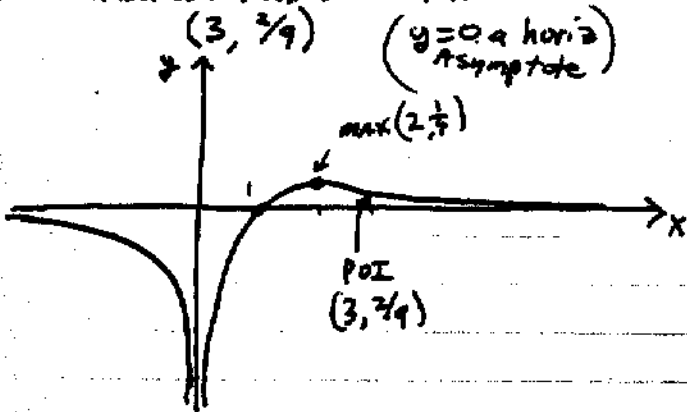
$x=0$ IS A vertical asymptote

x-intercept at (1, 0)

CRIT PT AT $x=2$

$f''(2) < 0 \Rightarrow$ LOCAL MAX (2, 1/4)

P.O.I. when $2x-6=0 \Rightarrow x=3$



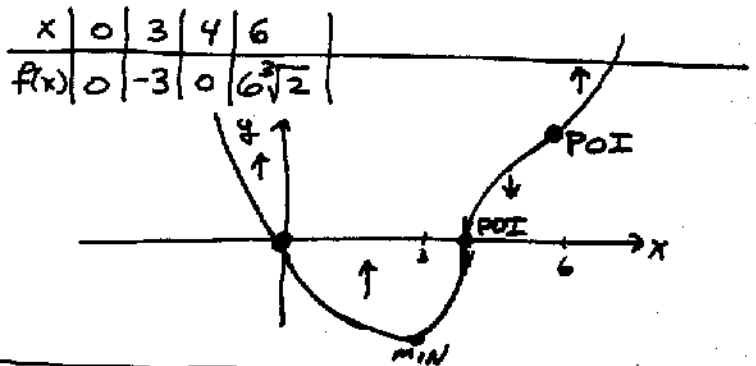
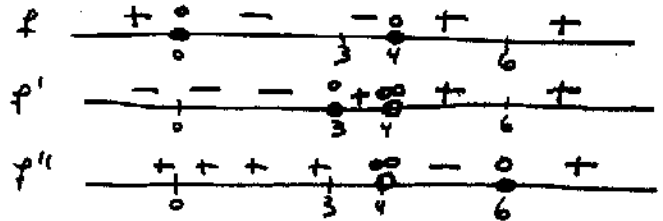
(24) $f(x) = x \sqrt[3]{x-4}$ $x \geq 0, x=4$ x-intercepts.

$f'(x) = \frac{4(x-3)}{3(x-4)^{2/3}}$ $x=3$ CRIT POINT

$x=4$ C.P., vert. tangent

$f''(x) = \frac{4(x-6)}{9(x-4)^{5/3}}$ $x=6$ PT. of inflect.

$x=4$ ALSO



(25)



$V = lwh = \frac{10}{3}$

$l = 2w$

$\Rightarrow 2w^2h = \frac{10}{3}$

Cost $C = [2 \cdot (2wh) + 2 \cdot wh + 2 \cdot w \cdot w] \cdot 2 + 2w^2(2)$

$C = (6wh + 2w^2) \cdot 2 + 6w^2$

$C = 1.2wh + w^2$ and $h = \frac{10}{6w^2} = \frac{5}{3w^2}$

$C(w) = 1.2w(\frac{5}{3w^2}) + w^2$

$C(w) = \frac{2}{w} + w^2$

$2w = \frac{2}{w^2}$

$C'(w) = -\frac{2}{w^2} + 2w = 0 \Rightarrow 2w^3 = 2$

$w = 1$

$\therefore l = 2, h = \frac{5}{3}$ ($C''(1) > 0 \Rightarrow$ Minimum)

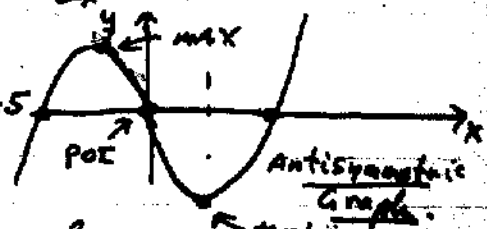
COST = 2 + 1 = \$3.00 per box.

(26) $y = 3x^{2/3} - 5x$ $\frac{dy}{dx} = 5x^{-1/3} - 5$

$\frac{d^2y}{dx^2} = \frac{10}{3x^{4/3}}$

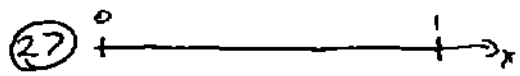
$y=0 \Rightarrow x=0$ OR $3x^{2/3} = 5$

$\Rightarrow x = \pm(\frac{5}{3})^{3/2}$

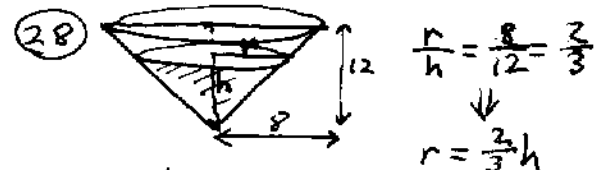


$\frac{dy}{dx} = 0 \Rightarrow x=1$ (1, -2) and

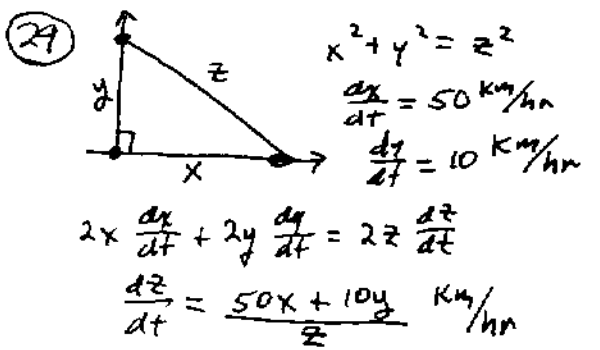
POI at $x=0$ $x=-1$ (-1, 2)



$a \text{ and } d = a = \text{constant}$
 $v(t) = at + C$
 $v(0) = 0 + C = 0$ (rest)
 $\therefore v(t) = at = \frac{dx}{dt}$
 $x(t) = \frac{1}{2}at^2 + C'$
 $x(0) = 0 + C' = 0$
 so $x(t) = \frac{1}{2}at^2$
 If time on runway = T ,
 then $v(T) = aT = 100 \text{ mi/hr}$
 $x(T) = \frac{1}{2}aT^2 = 1$
 $\Rightarrow \frac{1}{2}T(aT) = \frac{1}{2}T(100) = 1$
 $\Rightarrow T = \frac{1}{50} \text{ hr}$
 and $a = \frac{100}{T} = 5000 \text{ mi/hr}^2$



$V = \frac{1}{3}\pi r^2 h = \frac{4}{27}\pi h^3$
 $\frac{dV}{dt} = \frac{4}{9}\pi h^2 \frac{dh}{dt}$
 $\frac{dV}{dt} = -36 \text{ ft}^3/\text{min}, h = 9 \text{ ft}$
 $\Rightarrow -36 = \frac{4}{9}\pi \cdot 81 \frac{dh}{dt}$
 $\Rightarrow \frac{dh}{dt} = \frac{-36 \cdot 9}{4 \cdot 81 \pi} = \boxed{-\frac{1}{\pi} \text{ ft/min}}$



$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$
 $\frac{dz}{dt} = \frac{50x + 10y}{z} \text{ km/hr}$
 we can compute this with add'l info.
 For example after 1 hour, $x = 50, y = 40$
 and $z = 10\sqrt{26}$
 $\Rightarrow \frac{dz}{dt} = \frac{2500 + 100}{10\sqrt{26}} = 10\sqrt{26} \text{ km/hr}$
 But, RATIOS $\frac{x}{z} = \frac{5}{\sqrt{26}}$ and $\frac{y}{z} = \frac{4}{\sqrt{26}}$
 REMAIN CONSTANT, so $\frac{dz}{dt} = 10\sqrt{26} \text{ km/hr}$ always.

$v_0 = 200 \text{ ft/sec}$ $a = -10 \text{ ft/sec}^2$
 $v(t) = -10t + 200$ $x_0 = 0$
 $x(t) = -5t^2 + 200t$

when $x = 1280$, we have
 $-5t^2 + 200t = 1280$
 $5t^2 - 200t + 1280 = 0$
 $t = \frac{200 \pm \sqrt{40000 - 25600}}{10}$
 $= \frac{200 \pm 120}{10} = 32 \text{ sec or } \boxed{8 \text{ sec}}$
 $v(8) = -80 + 200 = 120 \text{ ft/sec}$ which exceeds the speed limit.

$\textcircled{31} \textcircled{a} \lim_{x \rightarrow \infty} 2x \ln(1 + \frac{3}{x}) \rightarrow \infty \cdot 0$
 $= \lim_{x \rightarrow \infty} \frac{2 \ln(1 + \frac{3}{x})}{\frac{1}{x}} \rightarrow \frac{0}{0}$
 $\therefore = \lim_{x \rightarrow \infty} \frac{2 \cdot \frac{-3/x^2}{1 + 3/x}}{(-1/x^2)} = \lim_{x \rightarrow \infty} \frac{6}{1 + 3/x} = \boxed{6}$

$\textcircled{b} \lim_{x \rightarrow 0} \frac{x e^x - x}{1 - \cos x} \rightarrow \frac{0}{0} \therefore = \lim_{x \rightarrow 0} \frac{x e^x + e^x - 1}{\sin x} \rightarrow \frac{0}{0}$
 $\therefore = \lim_{x \rightarrow 0} \frac{x e^x + 2e^x}{\cos x} = \frac{2}{1} = \boxed{2}$

$\textcircled{c} L = \lim_{x \rightarrow 0} (1 + \sin 2x)^{1/x} \rightarrow 1^\infty$ indeterminate
 $\ln L = \lim_{x \rightarrow 0} \frac{\ln(1 + \sin 2x)}{x} \rightarrow \frac{0}{0}$ $\ln L = 2$
 $\therefore = \lim_{x \rightarrow 0} \frac{2 \cos 2x}{1 + \sin 2x} = \frac{2}{1} = 2$ so $L = \boxed{e^2}$

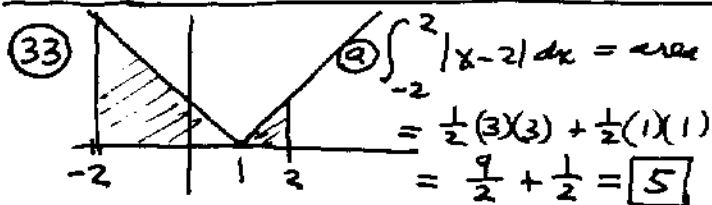
$\textcircled{d} \lim_{x \rightarrow 0} \frac{\sec x - 1}{x^2} \rightarrow \frac{0}{0} \therefore = \lim_{x \rightarrow 0} \frac{\sec x \tan x}{2x}$
 $= \lim_{x \rightarrow 0} \frac{\sin x}{2(\cos x)^2 \cdot x} = \lim_{x \rightarrow 0} \left(\frac{1}{2(\cos x)^2} \right) \cdot \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)$
 $= \frac{1}{2} \cdot 1 = \boxed{\frac{1}{2}}$

\textcircled{e} For $x \neq 0, \sin x \approx x, \sin(x^2) \approx x^2$
 so $\frac{x \sin x}{\sin(x^2)} \approx \frac{x \cdot x}{x^2} = 1 \therefore \lim_{x \rightarrow 0} \frac{x \sin x}{\sin(x^2)} = 1$

$\textcircled{f} \lim_{x \rightarrow 0} \frac{x e^x - x}{e^x - x - 1} \rightarrow \frac{0}{0} \therefore = \lim_{x \rightarrow 0} \frac{x e^x + e^x - 1}{e^x - 1} \rightarrow \frac{0}{0}$
 $\therefore = \lim_{x \rightarrow 0} \frac{x e^x + 2e^x}{e^x} = \frac{2}{1} = \boxed{2}$

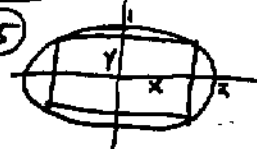
32) $L = \lim_{x \rightarrow \infty} X^{2/x} \rightarrow \infty^0$ (Indet.)
 $\ln L = \lim_{x \rightarrow \infty} \ln(X^{2/x}) = \lim_{x \rightarrow \infty} \left(\frac{2 \ln X}{x} \right) \rightarrow \frac{\infty}{\infty}$
 $\therefore = \lim_{x \rightarrow \infty} \left(\frac{2/x}{1} \right) = 0$ $\ln L = 0$ So $L = e^0 = 1$

33) Write $y = X^{2/x}$ use log. diff'n.
 $\ln y = \ln X^{2/x} = \frac{2 \ln X}{x}$
 $\frac{1}{y} \frac{dy}{dx} = \frac{X \cdot \frac{2}{x} - 2 \ln X}{x^2} = \frac{2 - 2 \ln X}{x^2}$
 So $\frac{dy}{dx} = \frac{2(1 - \ln X)}{x^2} X^{2/x}$



33) $\bar{f} = \frac{1}{5} \int_0^5 |x-2| dx = \frac{1}{5} \left[\frac{1}{2}(2 \cdot 2) + \frac{1}{2}(3 \cdot 3) \right]$
 $= \frac{1}{5} \cdot \frac{13}{2} = \frac{13}{10} = 1.3$

34) $v_0 = 25 \text{ m/s}$ $a = -5 \text{ m/s}^2$
 $v(t) = -5t + 25$ $x_0 = 0$
 $x(t) = -\frac{5}{2}t^2 + 25t$
 STOPPED when $v=0 \Rightarrow -5t + 25 = 0$
 so $t = 5$
 $x(5) = -\frac{5}{2}(25) + 125 = \frac{125}{2} = 62 \frac{1}{2} \text{ Meters}$

35)  $A = (2x)(2y) = 4xy$
 $\frac{x^2}{4} + y^2 = 1$
 $x = 2(1-y^2)^{1/2}$
 So $A(y) = 8y(1-y^2)^{1/2}$ $0 \leq y \leq 1$
 $A'(y) = (8y)^{1/2}(1-y^2)^{1/2}(-2y) + 8(1-y^2)^{1/2}$
 $= \frac{8(1-y^2) - 8y^2}{\sqrt{1-y^2}} = \frac{8-16y^2}{\sqrt{1-y^2}} = 0$
 $\Rightarrow 16y^2 = 8 \Rightarrow y^2 = \frac{1}{2}$
 $y = \frac{\sqrt{2}}{2} \Rightarrow x = \sqrt{2}$
 \Rightarrow Dimensions are $2\sqrt{2}$ by $\sqrt{2}$

36) $\int_{-3}^2 f(x) dx = \int_{-3}^0 (x+3) dx + \int_0^2 x^2 dx$
 $= \left[\frac{x^2}{2} + 3x \right]_{-3}^0 + \left[\frac{x^3}{3} \right]_0^2$
 $= 0 - \left(\frac{9}{2} - 9 \right) + \frac{8}{3} - 0$
 $= \frac{9}{2} + \frac{8}{3} = \frac{43}{6}$

37)

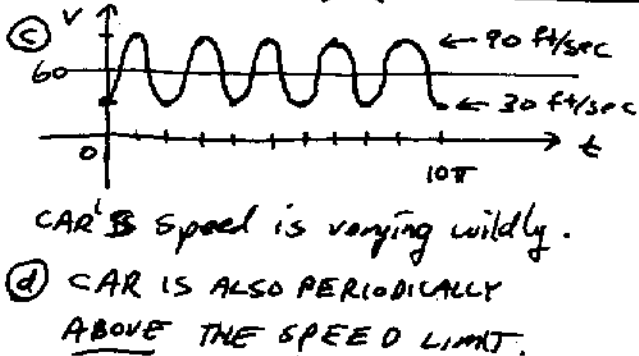
x	0	.5	1	1.5	2
f(x)	1	1.118	1.414	1.803	2.236

 $n=2 \Rightarrow \Delta x = 1$

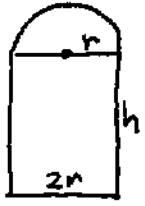
38) $L = 1(f(0) + f(1)) = 2.414$
 $R = 1(f(1) + f(2)) = 1.414 + 2.236 = 3.650$
 $T = \frac{L+R}{2} = 3.032$
 $M = 1(f(.5) + f(1.5)) = 2.921$
 (with $n=4$) $S = \frac{2M+T}{3} = 2.958$
 39) $L < M < S < T < R$

39) On $[0, 2]$, $f'(x) > 0 \Rightarrow$ Increasing.
 $f''(x) > 0 \Rightarrow$ concave up.
 Hence L underestimates, R overestimates, and T overestimates.

38) $a = 30 \sin t$ ft/sec^2 $v_0 = 30 \text{ ft/sec}$
 $\Rightarrow v(t) = -30 \cos t + C$
 $v(0) = -30 + C = 30 \Rightarrow C = 60$
 a) So $v(t) = -30 \cos t + 60$
 b) $x(3\pi) - x(0) = \int_0^{3\pi} v(t) dt = \int_0^{3\pi} (-30 \cos t + 60) dt$
 $= \int_0^{3\pi} (-30 \cos t + 60) dt$
 $= -30 \sin t + 60t \Big|_0^{3\pi}$
 $= (-30(0) + 180\pi) - (0 + 0) = 180\pi \text{ ft}$



39 $P = 2h + 2r + \pi r = 24$



$2h + (2 + \pi)r = 24$

MAXIMIZE AREA

$A = 2rh + \frac{1}{2}\pi r^2$

Solve for $h = \frac{24 - (2 + \pi)r}{2}$

$h = 12 - (1 + \frac{\pi}{2})r$

$A(r) = 24r - (2 + \pi)r^2 + \frac{\pi}{2}r^2$

$A'(r) = 24 - (2 + \frac{\pi}{2})r$

endpts: $r=0$ and $h=0$

$\Rightarrow r = \frac{24}{2 + \pi}$

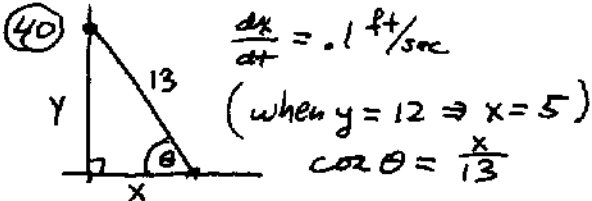
CRIT PT: $A'(r) = 24 - (2 + \frac{\pi}{2})r = 0$

$\Rightarrow r = \frac{24}{4 + \pi}$

$A(\frac{24}{4 + \pi}) = \frac{576}{4 + \pi} - \frac{(4 + \pi)(\frac{24}{4 + \pi})^2}{2}$
 $= \frac{288}{4 + \pi} \approx 40$

$A(\frac{24}{2 + \pi}) = \frac{\pi(\frac{24}{2 + \pi})^2}{2} = \frac{288\pi}{(2 + \pi)^2} \approx 34$

MAX area at CRITICAL POINT



$\frac{dx}{dt} = .1 \text{ ft/sec}$

(when $y = 12 \Rightarrow x = 5$)

$\cos \theta = \frac{x}{13}$

$-\sin \theta \frac{d\theta}{dt} = \frac{1}{13} \frac{dx}{dt}$

when $y = 12, x = 5 \Rightarrow \sin \theta = \frac{12}{13}$

$\Rightarrow -\frac{12}{13} \frac{d\theta}{dt} = \frac{1}{13} (.1)$

$\Rightarrow \frac{d\theta}{dt} = \frac{-.1}{12} = \frac{-1}{120} \text{ rad/sec}$

41 $s(t) = -2t^3 - 3t^2 + 12t + 40$

$v(t) = -6t^2 - 6t + 12$
 $= -6(t^2 + t - 2) \quad 0 \leq t \leq 3$

$v = 0$ when $(t+2)(t-1) = 0 \Rightarrow t = 1$

$v(0) = 12$ (forward) $v(1) = 0$ (stop)

$v(3) = -54 - 18 + 12 < 0$ (backward)

$s(0) = 40 \quad s(1) = -2 - 3 + 12 + 40 = 47$

$s(3) = -54 - 27 + 36 + 40 = -5$

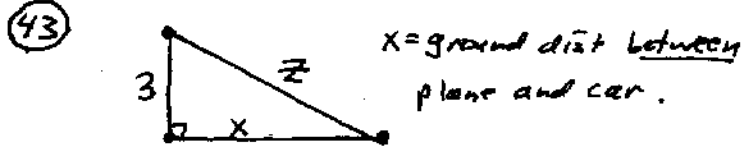
TOTAL DISTANCE = $|47 - 40| + |-5 - 47|$
 $= 7 + 52 = 59 \text{ units}$

42 $f(x) = x^3 e^{-x}$
 $f'(x) = -x^3 e^{-x} + 3x^2 e^{-x}$
 $= x^2 e^{-x} (3 - x)$
 $f''(x) = x^3 e^{-x} - 6x^2 e^{-x} + 6x e^{-x}$
 $= x e^{-x} (x^2 - 6x + 6)$

Decreasing where $f'(x) < 0 \Rightarrow x > 3$

Decreasing most rapidly (for $x > 3$) where $f''(x) = 0 \Rightarrow x = \frac{6 \pm \sqrt{36 - 24}}{2}$

$\Rightarrow x = 3 + \sqrt{3}$



$x^2 + 9 = z^2 \quad 2x \frac{dx}{dt} = 2z \frac{dz}{dt}$

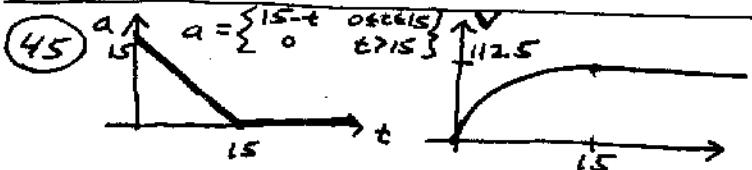
when $z = 5 \Rightarrow x = 4, \frac{dz}{dt} = -160 \text{ mi/hr}$

$\Rightarrow 4 \frac{dx}{dt} = (5)(-160) \quad \frac{dx}{dt} = -200 \text{ mi/hr}$

Now Adjust for ground speed of plane (120 mi/hr)

$\Rightarrow \text{car's velocity} = 200 - 120 = 80 \text{ mi/hr}$

44 $\int_0^1 f(1-x) dx = -\int_1^0 f(u) du$
 $u = 1-x \quad du = -dx = +\int_0^1 f(u) du$
 So $\int_0^1 f(1-x) dx = \int_0^1 f(x) dx$



on $[0, 15] \quad a = 15t - \frac{1}{2}t^2 + C \quad v(0) = C = 0$

$\Rightarrow v(t) = 15t - \frac{1}{2}t^2 \quad v(15) = 225 - \frac{225}{2} = \frac{225}{2}$

for $t > 15 \quad v(t) = \frac{225}{2} = 112.5 \text{ ft/sec}$
 $= \text{MAX. velocity}$

46 $\int_{-3}^2 f(x) dx = \int_{-3}^0 (x+3) dx + \int_0^2 x^2 dx$
 $= \frac{43}{6}$ (SAME PROBLEM AS #36 (OOPS!))

47 $x^2 - 4x + y^2 + 3 = 0$ (a)
 $x^2 - 4x + 4 + y^2 = -3 + 4$
 $(x-2)^2 + y^2 = 1$

LINE THROUGH (0,0) $\Rightarrow y = mx$ (b)
 $2x - 4 + 2y \frac{dy}{dx} = 0$
 $\Rightarrow \frac{dy}{dx} = \frac{4-2x}{2y} = \frac{2-x}{y} = m_1$ (c)

(a) and (b) $\Rightarrow x^2 - 4x + m^2 x^2 + 3 = 0$
 (b) and (c) $\Rightarrow m^2 x = 2 - x$
 So $x^2 - 4x + (2-x)x + 3 = 0$
 $x^2 - 4x + 2x - x^2 + 3 = 0$
 $3 - 2x = 0 \Rightarrow x = \frac{3}{2}$
 $\Rightarrow y^2 = 4x - x^2 - 3 = 6 - \frac{9}{4} - 3 = \frac{3}{4}$
 $\Rightarrow y = \pm \frac{\sqrt{3}}{2} \Rightarrow m = \pm \frac{\sqrt{3}}{2}$
 \Rightarrow Lines $y = \frac{\sqrt{3}}{2}x$ and $y = -\frac{\sqrt{3}}{2}x$

48 $f(x) = 2 \sin x - 3 \cos x$ $x_0 = \frac{3\pi}{2}$
 $f'(x) = 2 \cos x + 3 \sin x$
 $f(\frac{3\pi}{2}) = -2$ $f'(\frac{3\pi}{2}) = -3$
 $y = -2 - 3(x - \frac{3\pi}{2})$

49 $x^2 y + 2y^3 = 3x + 2y + 54$ (2,3)
 $2xy + x^2 \frac{dy}{dx} + 6y^2 \frac{dy}{dx} = 3 + 2 \frac{dy}{dx}$
 $(x^2 + 6y^2 - 2) \frac{dy}{dx} = 3 - 2xy$
 $\frac{dy}{dx} = \frac{3 - 2xy}{x^2 + 6y^2 - 2} = \frac{3 - 12}{4 + 54 - 2} = -\frac{9}{56}$
 TL is $y - 3 = -\frac{9}{56}(x - 2)$

50 $f(2) = 2, f'(2) < 1, f''(2) < 0$
 $\int_0^2 f'(x) dx = f(2) - f(0) = 2 - 0 = 2$
 $\int_0^2 f(x) dx > 0$
 $\Rightarrow 2 < \int_0^2 f(x) dx < 4$
 $f''(2) < 0 < f'(2) < 1 < f(2) = \int_0^2 f'(x) dx < \int_0^2 f(x) dx$

51
 $\frac{dx}{dt} = 220 \text{ ft/sec}$
 $\frac{x}{100} = \tan \theta$ $\frac{1}{100} \frac{dx}{dt} = \sec^2 \theta \frac{d\theta}{dt}$
 $\Rightarrow \frac{d\theta}{dt} = \frac{1}{100} \cos^2 \theta \frac{dx}{dt} = 2.2 \cos^2 \theta$
 (a) $x=0 \Rightarrow \theta=0 \Rightarrow \frac{d\theta}{dt} = 2.2 \text{ RAD/sec}$
 (b) $x=240 \Rightarrow \cos \theta = \frac{100}{\sqrt{(100)^2 + (240)^2}}$
 $\text{So } \frac{d\theta}{dt} = 2.2 \left[\frac{100^2}{(100)^2 + (240)^2} \right] = \frac{22000}{67600} = \frac{55}{169} \text{ RAD/sec}$
 $(\approx .325 \text{ RAD/sec})$

52
 $V = \pi r^2 h = Ah, A = \text{area}$
 $\frac{dV}{dt} = A \frac{dh}{dt} + h \frac{dA}{dt}$
 $\frac{dV}{dt} = -5 \text{ ft}^3/\text{hr}, r=500 \Rightarrow A = 250000\pi \text{ ft}^2, \frac{dh}{dt} = -.001$
 $\Rightarrow -5 = (250000\pi)(-.001) + (h) \frac{dA}{dt}$
 $\Rightarrow \frac{dA}{dt} = (25000\pi - 500) \text{ ft}^2/\text{hr}$ AREA INCREASING.

53
 $t_1 = \text{time in snow}$
 $t_2 = \text{time on road}$
 $3 \cdot t_1 = \sqrt{x^2 + 9}$ $t_2 = 6 - x$
 Total time $T = t_1 + t_2 = \frac{1}{3} \sqrt{x^2 + 9} + \frac{1}{4}(6 - x)$
 $\frac{dT}{dx} = \frac{x}{3\sqrt{x^2 + 9}} - \frac{1}{4} = 0 \Rightarrow x = \frac{9}{\sqrt{7}}$
 $T(\frac{9}{\sqrt{7}}) = \frac{1}{3} \sqrt{\frac{81}{7} + 9} + \frac{1}{4}(6 - \frac{9}{\sqrt{7}}) = 2.16 \text{ hrs.}$
 So, you'll get there before midnight.

54 (a) $F'(x) = \sin 2x + \cos 2x$ $F(0) = 0$
 $\Rightarrow F(x) = -\frac{1}{2} \cos 2x + \frac{1}{2} \sin 2x + C$
 $F(0) = -\frac{1}{2} + C = 0 \Rightarrow C = \frac{1}{2} \Rightarrow F(x) = -\frac{1}{2} \cos 2x + \frac{1}{2} \sin 2x + \frac{1}{2}$
 (b) $F'(x) = x^{3/2} - x^{5/2} \Rightarrow F(x) = \frac{2}{5} x^{5/2} - \frac{2}{7} x^{7/2} + C$
 $F(0) = 0 + C = 0 \Rightarrow C = 0 \Rightarrow F(x) = \frac{2}{5} x^{5/2} - \frac{2}{7} x^{7/2}$
 (c) $f'(x) = 2x - 3 \Rightarrow f(x) = x^2 - 3x + C$
 $f(1) = 1 - 3 + C = -2 = 0 \Rightarrow C = 2 \Rightarrow f(x) = x^2 - 3x + 2$
 $f(x) = 0 \Rightarrow x^2 - 3x + 2 = (x-2)(x-1) = 0$
 $\Rightarrow x = 1$ or $x = 2$