Last Name: _____

First Name: _____

Math 1a Exam #2: Tuesday, November 23, 1999

SECTION (CIRCLE ONE):

Bing Cheng	Bing Cheng	Robert Winters	Laura DeMarco
MWF 10	MWF 11	MWF 10	MWF 12
Manoj Viswanathan (CA)	Eiichi Miyasaka (CA)	Sahir Islam (CA)	Brian Shaffer (CA)

Lisa Carbone	Lisa Carbone	Grisha Mihalkin
TTh 10	TTh 11:30	TTh 11:30
Karen Acquista (CA)	Peter Hamel (CA)	Jacob Honoroff (CA)
Rodolfo Perez (CA)		

| Question	Points	Score
1	15	
2	14	
3	16	
4	14	
5	14	
6	15	
7	12	
Total	100	

The time allotted for this exam is 90 minutes.

Calculators are _not_ permitted.

Justify your answers carefully. No partial credit can be given for unsubstantiated answers.

(1) Consider the two small Zairian towns of Abedi and Bueya, which currently cannot be reached by road. The main road from Kinshasa to Matadi passes 10 km south of the two towns.

The government plans to build a road system connecting the two towns to the main road, as shown in the diagram. How far from the main road should the junction point be chosen to minimize the total length of the new roads?

Abedi ___ _10 km_ _ _ Bueya

roads of equal length

10 km   junction   10 km

main road
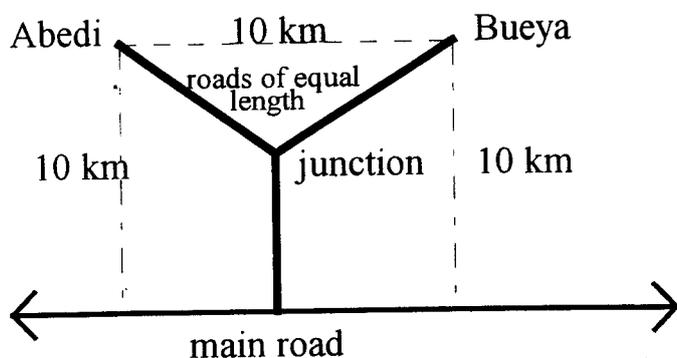
(2) A creature moves along the x-axis in such a way that its acceleration is given by $a(t) = 4 - 12t$. We also know that when $t = 1$ it is at the position $x = 3$ and has zero velocity at that moment. Where will the creature be when $t = 2$?

(3) A function $f(x)$ and its first and second derivatives are given below.

$$f(x) = \frac{x-1}{x^2 - 2x}$$

$$f'(x) = \frac{-x^2 + 2x - 2}{(x^2 - 2x)^2}$$

$$f''(x) = \frac{2(x-1)(x^2 - 2x + 4)}{(x^2 - 2x)^3}$$

a) Identify any x or y intercepts for the graph of this function.

b) Find any critical points and determine whether they are relative maxima or minima or neither.

c) Find any points of inflection.

d) Find any vertical asymptotes for this graph.

e) Find any horizontal asymptotes for this graph.

f) Give a rough sketch of the graph of this function, illustrating all of the features you found in parts a) through e).

$\uparrow y$

$\rightarrow x$
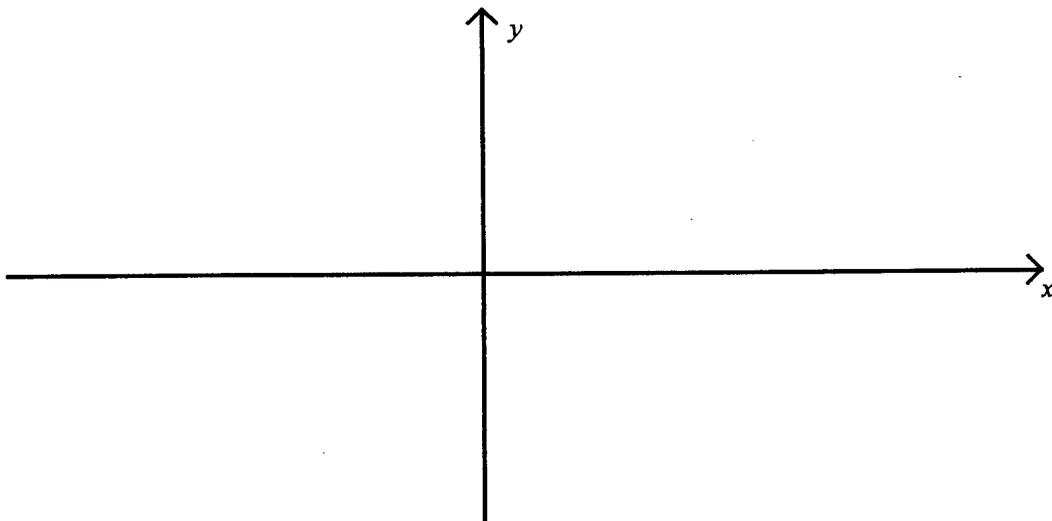
(4) A manufacturing plant has a capacity of 25 articles per week. Experience has shown that n articles can be sold at a price of p dollars each where $p = 110 - 2n$ and the cost of producing n articles is known to be $600 + 20n + n^2$ dollars. How many articles should be made each week to give the largest profit?

(5) A man 6 feet tall walks away from a source of light 20 feet above the ground at a speed of 3 feet per second. At what rate is the shadow of his head moving along the ground?

(6) Find

(a) $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x^3}$

(b) $\lim\limits_{x \to +\infty} \dfrac{e^{2x}}{3x^2 + 1}$

(c) $\lim\limits_{x \to 1} \dfrac{x^2 \ln x}{x^3 - 1}$

(7) Find the point on the parabola $y = x^2$ that is closest to the point (4, 1/2).