Math 1A Midterm #1 Review Sheet

tangents: a line which touches the curve once at point P

slope of tangent: $m = \lim_{Q \to P} m_{PQ}$

limits $\lim_{x \to a} f(x) = L$ is read as:

"the limit of $f(x)$, as x approaches a, equals L"

one-sided limits: $\lim_{x \to a^+} f(x) = L$ → only consider $x > a$

$\lim_{x \to a^-} f(x) = L$ → only consider $x < a$

NOTE: $\lim_{x \to a} f(x) = L$ if & only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

limit laws: given: c is a constant; n is pos. integer; limits $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist

1. $\lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a} (f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a} (c f(x)) = c \lim_{x \to a} f(x)$
4. $\lim_{x \to a} (f(x) g(x)) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
5. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$
6. $\lim_{x \to a} (f(x))^n = (\lim_{x \to a} f(x))^n$
7. $\lim_{x \to a} c = c$
8. $\lim_{x \to a} x = a$
9. $\lim_{x \to a} x^n = a^n$
10. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ (if n is even, assume $a > 0$)

more generally: $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$

Direct Sub Property: if f is polynomial or rational function and "a" is in the domain of f,

$\Rightarrow \lim_{x \to a} f(x) = f(a) \Leftarrow$

Theorem: if $f(x) \leq g(x)$ when x is near "a" and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$

Squeeze Theorem: if $f(x) \leq g(x) \leq h(x)$ when x is near "a" and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$

Continuity: a function f is continuous at "a" if $\lim_{x \to a} f(x) = f(a)$ or if f is differentiable at "a"

likewise, a function f is continuous on an interval if it is continuous at every number in the interval

Theorem: if f & g are continuous at "a" and "c" is a constant, then the following functions are also continuous at "a": ① $f+g$ ② $f-g$ ③ cf ④ fg ⑤ f/g if $g(a) \neq 0$

Theorem: $\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} (g(x)))$

Vertical Asymptotes: the line $x = a$ is a vertical asymptote of the curve $y = f(x)$ if one of the combinations is true: $\lim_{x \to a^\pm} f(x) = \pm \infty$

LIMITS @ ∞

$$\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}, \quad \lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$$

if n is pos. integer,

$$\lim_{x \to \pm \infty} \frac{1}{x^n} = 0 \qquad \lim_{x \to -\infty} e^x = 0$$

Tangents (again)

⇒ the tangent to the curve $y = f(x)$ at the point $P(a, f(a))$ has slope:

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

DERIVATIVES of function f at "a":

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

where $h = x - a$

(conceptually: $f'(a)$ is the instant. rate of change of $y = f(x)$ w/ respect to x when $x = a$

NOTE: "a" can be replaced by variable x

acceleration $a(t) = v'(t) = s''(t)$

if $f''(x) > 0$, f is concave upward

if $f''(x) < 0$, f is concave downward