

8. (12 points) State the Intermediate Value Theorem and use it to show that there is a root of the equation $\cos x = x$ in the interval $(0, \frac{\pi}{2})$.

I.V.T. - IF $f(x)$ IS CONTINUOUS ON $[A, B]$
AND η IS A NUMBER SATISFYING
 $f(A) < \eta < f(B)$, THERE IS A NUMBER
 c IN (A, B) FOR WHICH $f(c) = \eta$.

LET $f(x) = x - \cos x$.

$f(x)$ IS CONTINUOUS ON $[0, \frac{\pi}{2}]$ AND

$$f(0) = -1 < 0 < \frac{\pi}{2} = f\left(\frac{\pi}{2}\right)$$

THUS, BY I.V.T. THERE IS SOME c IN $(0, \frac{\pi}{2})$

FOR WHICH $f(c) = 0$.

$$\text{I.E. } c - \cos c = 0$$

$$\text{I.E. } c = \cos c \quad \blacksquare$$