

4. (12 points) Answer true or false. Do not justify your answers.

- (i) If  $f(x)$  has a vertical asymptote at  $x = a$ , then  $\lim_{x \rightarrow a^+} f(x)$  does not exist. **F**
- (ii) If  $\lim_{x \rightarrow a^+} f(x)$  and  $\lim_{x \rightarrow a^-} f(x)$  exist, then  $f(x)$  is continuous at  $x = a$ . **F**
- (iii) A function may have at most two horizontal tangents. **F**
- (iv) If  $f(x)$  is differentiable at  $x = a$ , then  $\lim_{x \rightarrow a} f(x)$  exists. **T**
- (v) If  $\lim_{x \rightarrow \infty} f(x) = 4$ , then  $\lim_{x \rightarrow \infty} \frac{f(x)}{x} = 0$ . **T**
- (vi) If the graph of  $f(x)$  has a tangent line at  $(a, f(a))$ , then  $f'(a)$  exists. **F**
- (vii) If  $f(x)$  has a removable discontinuity at  $x = a$ , then  $\lim_{x \rightarrow a} f(x)$  exists. **T**
- (viii) An inflection point of a function is a point where its second derivative equals 0. **F**