

7. (14 points) Find the maximum possible area for the following window given that its perimeter must be 20 feet.



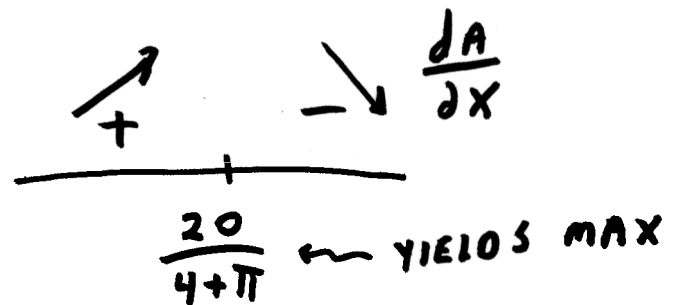
A RECTANGLE CAPPED BY A SEMICIRCLE.
(DOTTED SEGMENT IS NOT PART OF THE PERIMETER)

$$A = 2xy + \frac{1}{2}\pi x^2 \quad \& \quad P = 2y + 2x + \pi x = 20$$

$$\Rightarrow A = 2x \left(10 - x - \frac{\pi}{2}x\right) + \frac{1}{2}\pi x^2 \quad 0 < x < \frac{10}{1 + \frac{\pi}{2}}$$

$$\frac{dA}{dx} = 20 - 4x - \pi x = 0$$

$$\Rightarrow x = \frac{20}{4 + \pi}$$



THUS, THE MAXIMUM AREA

$$\frac{40}{4 + \pi} \left(10 - \frac{20}{4 + \pi} - \frac{10\pi}{4 + \pi}\right) + \frac{1}{2}\pi \left(\frac{20}{4 + \pi}\right)^2$$