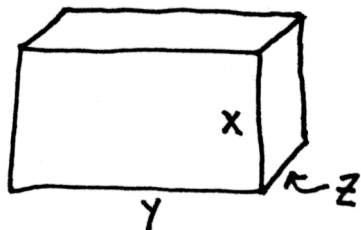


4. (14 points) Consider the box shown below when answering the following questions:

- (i) At what rate is the volume of the box changing if  $x = 10$ ,  $y = 5$ ,  $z = 2$  and  $\frac{dx}{dt} = -1$ ,  $\frac{dy}{dt} = 1$ ,  $\frac{dz}{dt} = 2$ ?
- (ii) If  $x$  and  $y$  are increasing at a rate of 2 units per minute, at what rate must  $z$  decrease in order to maintain a constant volume? (Your answer will be in terms of  $x$ ,  $y$  and  $z$ )



$$V = xyz \Rightarrow \frac{dV}{dt} = x \frac{d}{dt}(yz) + yz \frac{dx}{dt}$$

$$= x \left( y \frac{dz}{dt} + z \frac{dy}{dt} \right) + yz \frac{dx}{dt}$$

PLUGGING IN THE DATA YIELDS

$$\frac{dV}{dt} = 110$$

$$\text{SET } \frac{dV}{dt} = 0 \text{ WITH } \frac{dx}{dt} = \frac{dy}{dt} = 2$$

$$\text{TO GET } \frac{dz}{dt} = \frac{2xz + 2yz}{xy}$$

SO  $z$  MUST DECREASE AT A RATE OF

$$\frac{2xz + 2yz}{xy} \text{ UNITS MIN.}$$