

3. (12 points) Compute the following limits:

$$(i) \lim_{x \rightarrow 1} \frac{x^{10} - 1}{x^7 - 1} = \frac{10}{7} \quad \leftarrow \text{L'HOSPITAL}$$

$$(ii) \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1 \quad \leftarrow$$

$$(iii) \lim_{x \rightarrow 0^+} \frac{\cos x}{x} = \infty$$

$$(iv) \lim_{x \rightarrow 0^+} (1 + \sin x)^{1/x} = \lim_{x \rightarrow 0^+} e^{\frac{1}{x} \ln(1 + \sin x)}$$

$$\text{AND } \lim_{x \rightarrow 0^+} \frac{\ln(1 + \sin x)}{x}$$

$$\stackrel{*}{=} \lim_{x \rightarrow 0^+} \frac{\cos x}{1 + \sin x} = 1$$

$$\Rightarrow \text{ORIGINAL LIMIT} = e^1 = e$$