Math 1A Fall 2001: First Math 1A Exam, Fall 1998

1) Use the definition of the derivative, i.e. the limit of the difference quotient, to show that if $f(x) = \frac{1}{x^2}$, then $f'(x) = -\frac{2}{x^3}$.

2) Find the indicated derivatives:

a) Find $\frac{dy}{dx}$ if $y = \tan^2 x \sqrt{1+\pi^2}$

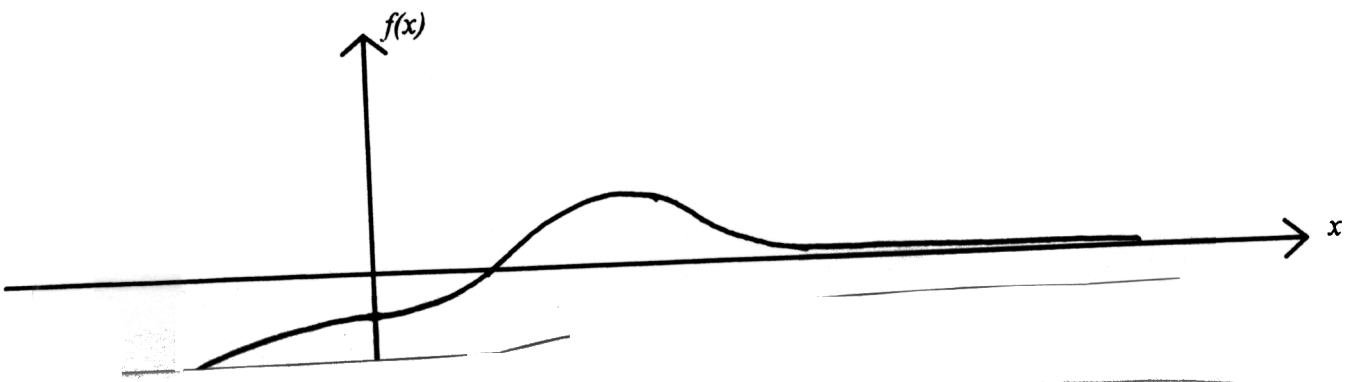
b) Find $\frac{dy}{dx}$ if $y = \cos^4(x^5)$

c) Find $f'(x)$ if $f(x) = \sin^2 3x + \cos^2 3x$

d) Find $h'(-1)$ if $h(x) = \frac{x^2 \ln(3x+4)}{x^3+2}$

e) Find $g'(t)$ if $g(t) = \pi t^3 + 3^{\pi t} + \pi^3 t$.

3) Given the graph of the function $y = f(x)$ as shown, graph the first and second derivatives on the given axes, marking all noteworthy points appropriately.



4) Suppose that $f(x)$ is a differentiable function whose derivative satisfies $f'(x) = (x^2-1)^{-1/2}$. Find $\frac{d}{dx}[f(\sec(x))]$ for $x \in (0, \frac{\pi}{2})$.

5) Find $\frac{dy}{dx}$ at the point (0,-1) for the relation $e^{(y^2+y)} = xy+1$.

6) a) Use the idea of linear approximation for the function $f(x) = \sqrt{x}$ and fact that $(2.5)^2 = 6.25$ to approximate $\sqrt{6}$.

b) Is this approximation less than or greater than the actual value of $\sqrt{6}$? Why?

7) A road runs at a right angle to a wall. There is a lamp on the ground 5 meters from the road and 10 meters from the wall. Fritz the Cat is walking on the road towards the wall. Find the rate at which Fritz's shadow is moving along the wall at the instant when he is 5 meters away from the wall, moving at 1 meter per second.