Math 1A Fall 2001: Section 3.2 Solutions

2. Quotient Rule: $F(x) = \dfrac{x - 3x\sqrt{x}}{\sqrt{x}} = \dfrac{x - 3x^{3/2}}{x^{1/2}} \Rightarrow$

$$F'(x) = \frac{x^{1/2}\left(1 - \frac{9}{2}x^{1/2}\right) - \left(x - 3x^{3/2}\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}\right)^2}$$

$$= \frac{x^{1/2} - \frac{9}{2}x - \frac{1}{2}x^{1/2} + \frac{3}{2}x}{x} = \frac{\frac{1}{2}x^{1/2} - 3x}{x} = \frac{1}{2}x^{-1/2} - 3$$

Simplifying first: $F(x) = \dfrac{x - 3x\sqrt{x}}{\sqrt{x}} = \sqrt{x} - 3x = x^{1/2} - 3x \Rightarrow F'(x) = \frac{1}{2}x^{-1/2} - 3$ (equivalent)

For this problem, simplifying first seems to be the better method.

6. By the Quotient Rule, $y = \dfrac{e^x}{1+x} \Rightarrow y' = \dfrac{(1+x)\,e^x - e^x(1)}{(1+x)^2} = \dfrac{e^x + xe^x - e^x}{(x+1)^2} = \dfrac{xe^x}{(x+1)^2}$.

12. $y = \dfrac{t^3 + t}{t^4 - 2} \Rightarrow y' = \dfrac{(t^4 - 2)(3t^2 + 1) - (t^3 + t)(4t^3)}{(t^4 - 2)^2} = \dfrac{(3t^6 + t^4 - 6t^2 - 2) - (4t^6 + 4t^4)}{(t^4 - 2)^2}$

$$= \frac{-t^6 - 3t^4 - 6t^2 - 2}{(t^4 - 2)^2} = -\frac{t^6 + 3t^4 + 6t^2 + 2}{(t^4 - 2)^2}$$

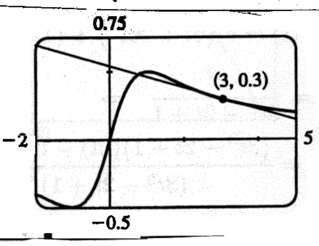
16. $z = w^{3/2}(w + ce^w) = w^{5/2} + cw^{3/2}e^w \Rightarrow$

$$z' = \tfrac{5}{2}w^{3/2} + c\left(w^{3/2} \cdot e^w + e^w \cdot \tfrac{3}{2}w^{1/2}\right) = \tfrac{5}{2}w^{3/2} + \tfrac{1}{2}cw^{1/2}e^w(2w + 3)$$

22. (a) $y = f(x) = \dfrac{x}{1+x^2} \Rightarrow$

$f'(x) = \dfrac{(1+x^2)1 - x(2x)}{(1+x^2)^2} = \dfrac{1 - x^2}{(1+x^2)^2}$. So the slope of the tangent line at the point $(3, 0.3)$ is $f'(3) = \frac{-8}{100}$ and its equation is $y - 0.3 = -0.08(x - 3)$ or $y = -0.08x + 0.54$.

(b)



28. We are given that $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$, and $g'(3) = 5$.

(a) $(f + g)'(3) = f'(3) + g'(3) = -6 + 5 = -1$

(b) $(fg)'(3) = f(3)g'(3) + g(3)f'(3) = (4)(5) + (2)(-6) = 20 - 12 = 8$

(c) $\left(\dfrac{f}{g}\right)'(3) = \dfrac{g(3)f'(3) - f(3)g'(3)}{[g(3)]^2} = \dfrac{(2)(-6) - (4)(5)}{(2)^2} = \dfrac{-32}{4} = -8$

(d) $\left(\dfrac{f}{f-g}\right)'(3) = \dfrac{[f(3) - g(3)]\,f'(3) - f(3)\,[f'(3) - g'(3)]}{[f(3) - g(3)]^2}$

$$= \frac{(4-2)(-6) - 4(-6-5)}{(4-2)^2} = \frac{-12 + 44}{2^2} = 8$$

32. (a) $y = x^2 f(x) \Rightarrow y' = x^2 f'(x) + f(x)(2x)$

(b) $y = \dfrac{f(x)}{x^2} \Rightarrow y' = \dfrac{x^2 f'(x) - f(x)(2x)}{(x^2)^2} = \dfrac{xf'(x) - 2f(x)}{x^3}$

(c) $y = \dfrac{x^2}{f(x)} \Rightarrow y' = \dfrac{f(x)(2x) - x^2 f'(x)}{[f(x)]^2}$

(d) $y = \dfrac{1 + xf(x)}{\sqrt{x}} \Rightarrow$

$$y' = \frac{\sqrt{x}\,[xf'(x) + f(x)] - [1 + xf(x)]\,\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$

$$= \frac{x^{3/2}f'(x) + x^{1/2}f(x) - \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{1/2}f(x)}{x} \cdot \frac{2x^{1/2}}{2x^{1/2}} = \frac{xf(x) + 2x^2 f'(x) - 1}{2x^{3/2}}$$

34. (a) $f(20) = 10{,}000$ means that when the price of the fabric is \$20/yard, 10,000 yards will be sold. $f'(20) = -350$ means that as the price of the fabric increases past \$20/yard, the amount of fabric which will be sold is decreasing at a rate of 350 yards per (dollar per yard).

(b) $R(p) = pf(p) \Rightarrow R'(p) = pf'(p) + f(p) \cdot 1 \Rightarrow$
$R'(20) = 20f'(20) + f(20) \cdot 1 = 20(-350) + 10{,}000 = 3000$. This means that as the price of the fabric increases past \$20/yard, the total revenue is increasing at \$3000/(\$/yard). Note that the Product Rule indicates that we will lose \$7000/(\$/yard) due to selling less fabric, but that that loss is more than made up for by the additional revenue due to the increase in price.

38. $y = \dfrac{x-1}{x+1} \Rightarrow y' = \dfrac{(x+1)(1) - (x-1)(1)}{(x+1)^2} = \dfrac{2}{(x+1)^2}$. If the tangent intersects the curve when $x = a$, then its slope is $2/(a+1)^2$. But if the tangent is parallel to $x - 2y = 2$, that is, $y = \frac{1}{2}x - 1$, then its slope is $\frac{1}{2}$. Thus, $\dfrac{2}{(a+1)^2} = \dfrac{1}{2} \Rightarrow$ $(a+1)^2 = 4 \Rightarrow a + 1 = \pm 2 \Rightarrow a = 1$ or -3. When $a = 1$, $y = 0$ and the equation of the tangent is $y - 0 = \frac{1}{2}(x - 1)$ or $y = \frac{1}{2}x - \frac{1}{2}$. When $a = -3$, $y = 2$ and the equation of the tangent is $y - 2 = \frac{1}{2}(x + 3)$ or $y = \frac{1}{2}x + \frac{7}{2}$.