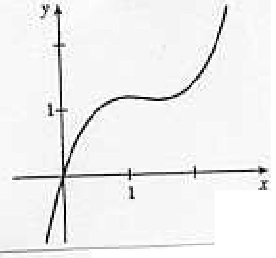
Math 1A Fall 2001: Section 2.7 Solutions

2. As h decreases, the line PQ becomes steeper, so its slope increases. So $0 < \frac{f(4)-f(2)}{4-2} < \frac{f(3)-f(2)}{3-2} < \lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$. Thus, $0 < \frac{1}{2}[f(4)-f(2)] < f(3)-f(2) < f'(2)$.

6.



8. Using Definition 2 with $g(x) = 1 - x^3$ and the point $(0,1)$, we have

$$g'(0) = \lim_{h\to 0}\frac{g(0+h)-g(0)}{h} = \lim_{h\to 0}\frac{[1-(0+h)^3]-1}{h} = \lim_{h\to 0}\frac{(1-h^3)-1}{h} = \lim_{h\to 0}(-h^2) = 0.$$

So an equation of the tangent line is $y - 1 = 0(x-0)$ or $y = 1$.

14. $$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{[(a+h)^4 - 5(a+h)] - (a^4 - 5a)}{h}$$
$$= \lim_{h\to 0}\frac{(a^4 + 4a^3h + 6a^2h^2 + 4ah^3 + h^4 - 5a - 5h) - (a^4 - 5a)}{h}$$
$$= \lim_{h\to 0}\frac{4a^3h + 6a^2h^2 + 4ah^3 + h^4 - 5h}{h} = \lim_{h\to 0}\frac{h(4a^3 + 6a^2h + 4ah^2 + h^3 - 5)}{h}$$
$$= \lim_{h\to 0}(4a^3 + 6a^2h + 4ah^2 + h^3 - 5) = 4a^3 - 5$$

22. By Equation 3, $\lim_{x\to\pi/4}\frac{\tan x - 1}{x - \pi/4} = f'(\pi/4)$, where $f(x) = \tan x$.

30. (a) $f'(8)$ is the rate of change of the quantity of coffee sold with respect to the price per pound when the price is \$8 per pound. The units for $f'(8)$ are pounds/(dollars/pound).

(b) $f'(8)$ is negative since the quantity of coffee sold will decrease as the price charged for it increases. People are generally less willing to buy a product when its price increases.

32. (a) $S'(T)$ is the rate of change of the maximum sustainable speed of Coho salmon with respect to the temperature. Its units are (cm/s)/°C.

(b) For $T = 15$ °C, it appears the tangent line to the curve goes through the points $(10, 25)$ and $(20, 32)$. So $S'(15) \approx \frac{32-25}{20-10} = 0.7$ (cm/s)/°C. This tells us that at $T = 15$ °C, the maximum sustainable speed of Coho salmon is changing at a rate of 0.7 (cm/s)/°C. In a similar fashion for $T = 25$ °C, we can use the points $(20, 35)$ and $(25, 25)$ to obtain $S'(25) \approx \frac{25-35}{25-20} = -2$ (cm/s)/°C. As it gets warmer than 20 °C, the maximum sustainable speed decreases rapidly.