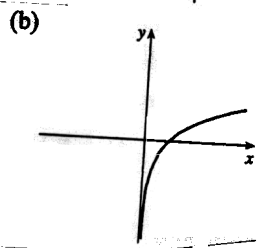
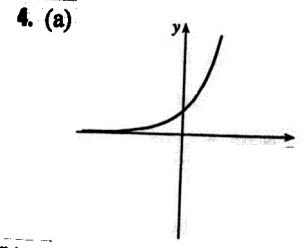
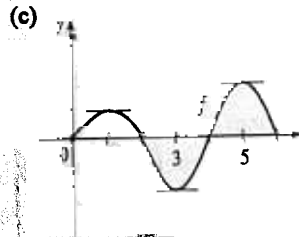


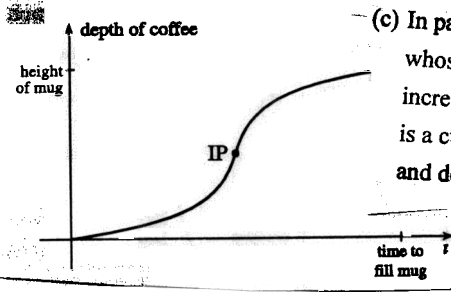
Math 1A Fall 2001: Section 2.10 Solutions

2. (a) $f'(x) > 0$ and f is increasing on $(0, 1)$ and $(3, 5)$. $f'(x) < 0$ and f is decreasing on $(1, 3)$ and $(5, 6)$.
- (b) Since $f'(x) = 0$ at $x = 1$ and $x = 5$ and f' changes from positive to negative at both values, f changes from increasing to decreasing and has local maxima at $x = 1$ and $x = 5$. Since $f'(x) = 0$ at $x = 3$ and f' changes from negative to positive there, f changes from decreasing to increasing and has a local minimum at $x = 3$.



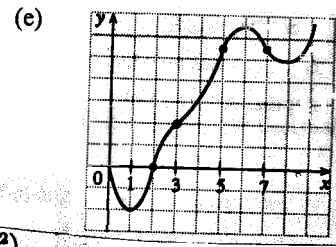
8. (a) If the position function is increasing, then the particle is moving toward the right. This occurs on t -intervals $(0, 2)$ and $(4, 6)$. If the function is decreasing, then the particle is moving toward the left—that is, on $(2, 4)$.
- (b) The acceleration is the second derivative and is positive where the curve is concave upward. This occurs on $(3, 6)$. The acceleration is negative where the curve is concave downward—that is, on $(0, 3)$.

10. At first the depth increases slowly because the base of the mug is wide. But as the mug narrows, the coffee rises more quickly. Thus, the depth d increases at an increasing rate and its graph is concave upward. The rate of increase of d has a maximum where the mug is narrowest; that is, when the mug is half full. It is there that the inflection point (IP) occurs. Then the rate of increase of d starts to decrease as the mug widens and the graph becomes concave down.



(c) In part (a), the graph of $y = e^x$ is a curve whose slope is always positive and increasing. In part (b), the graph of $y = \ln x$ is a curve whose slope is always positive and decreasing.

12. (a) f is increasing where f' is positive, on $(1, 6)$ and $(8, \infty)$, and decreasing where f' is negative, on $(0, 1)$ and $(6, 8)$.
- (b) f has a local maximum where f' changes from positive to negative, at $x = 6$, and local minima where f' changes from negative to positive, at $x = 1$ and at $x = 8$.
- (c) f is concave upward where f' is increasing, that is, on $(0, 2)$, $(3, 5)$, and $(7, \infty)$, and concave downward where f' is decreasing, that is, on $(2, 3)$ and $(5, 7)$.



(d) There are points of inflection where f changes its direction of concavity, at $x = 2$, $x = 3$, $x = 5$ and $x = 7$.

$$24. (a) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{[(x+h)^4 - 2(x+h)^2] - (x^4 - 2x^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - 2x^2 - 4xh - 2h^2) - (x^4 - 2x^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4 - 4xh - 2h^2}{h}$$

$$= \lim_{h \rightarrow 0} (4x^3 + 6x^2h + 4xh^2 + h^3 - 4x - 2h) = 4x^3 - 4x$$

$$f''(x) = \lim_{h \rightarrow 0} \frac{f'(x+h) - f'(x)}{h} = \lim_{h \rightarrow 0} \frac{[4(x+h)^3 - 4(x+h)] - (4x^3 - 4x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(4x^3 + 12x^2h + 12xh^2 + 4h^3 - 4x - 4h) - (4x^3 - 4x)}{h}$$

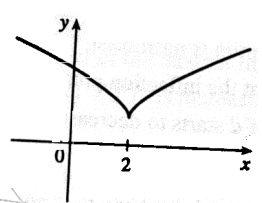
$$= \lim_{h \rightarrow 0} \frac{12x^2h + 12xh^2 + 4h^3 - 4h}{h}$$

$$= \lim_{h \rightarrow 0} (12x^2 + 12xh + 4h^2 - 4) = 12x^2 - 4$$

(b) $f'(x) > 0 \Leftrightarrow 4x^3 - 4x > 0 \Leftrightarrow 4x(x^2 - 1) > 0 \Leftrightarrow 4x(x+1)(x-1) > 0$, so f is increasing on $(-1, 0)$ and $(1, \infty)$ and f is decreasing on $(-\infty, -1)$ and $(0, 1)$.

(c) $f''(x) > 0 \Leftrightarrow 12x^2 - 4 > 0 \Leftrightarrow 12x^2 > 4 \Leftrightarrow x^2 > \frac{1}{3} \Leftrightarrow |x| > \sqrt{\frac{1}{3}}$, so f is CU on $(\sqrt{\frac{1}{3}}, \infty)$ and f is CD on $(-\infty, -\sqrt{\frac{1}{3}})$ and $(\sqrt{\frac{1}{3}}, \infty)$.

16. $f''(x) < 0$ on $(-\infty, 2)$ and $(2, \infty)$, so the graph of f is CD on these intervals. Since f is not differentiable at 2, f could be discontinuous at 2 or have a cusp or corner at 2.



28. The position function is the antiderivative of the velocity function, so its graph will have be horizontal where the velocity function is equal