Key

Math 19 Problem Set 8: p. 213-215 Ex. 1, 3, 5-8.

(7) $\frac{du}{dt} = -3\frac{du}{dx} - ru$, $r = 2$

1. $u(t,x) = e^{-2t}\sin(x-3t)$

$\frac{du}{dt} = -2e^{-2t}\sin(x-3t) - 3e^{-2t}\cos(x-3t)$

$\frac{du}{dx} = e^{-2t}\cos(x-3t)$

$-3\frac{du}{dx} - ru = -3e^{-2t}\cos(x-3t) - 2e^{-2t}\sin(x-3t)$

$= \frac{du}{dt}$

3. $u(t,x) = e^{-2t}e^{-2(x-3t)/3} = e^{-2t-\frac{2}{3}x+2t} = e^{-\frac{2}{3}x}$ ← $u(t,x) = e^{-2x}$ is not a solution (the book is wrong)

$\frac{du}{dt} = 0$ $\frac{du}{dx} = -\frac{2}{3}e^{-\frac{2}{3}x}$

$-3\frac{du}{dx} - ru = 2e^{-\frac{2}{3}x} - 2e^{-\frac{2}{3}x} = 0 = \frac{du}{dt}$

5. $u(t,x) = e^{-2t}(1+(x-3t)^2)^{-1}$

$\frac{du}{dt} = -2e^{-2t}(1+(x-3t)^2)^{-1} + e^{-2t}\frac{-2(x-3t)(-3)}{(1+(x-3t)^2)^2}$

$= -2e^{-2t}(1+(x-3t)^2)^{-1} + 6(x-3t)e^{-2t}(1+(x-3t)^2)^{-2}$

$\frac{du}{dx} = e^{-2t}(-1)(1+(x-3t)^2)^{-2}(2)(x-3t)$

$= -2(x-3t)e^{-2t}(1+(x-3t)^2)^{-2}$

$-3\frac{du}{dx} - ru = 6(x-3t)e^{-2t}(1+(x-3t)^2)^{-2} - 2e^{-2t}(1+(x+3t)^2)^{-1}$

$= \frac{du}{dt}$

(8) $u(t,x) = e^{-rt}f(x-3t)$, $r = 2$

6. $u(0,x) = \cos x$

$u(0,x) = e^{-2(0)}f(x-3\cdot 0) = f(x) = \cos x$

$u(t,x) = e^{-2t}\cos(x-3t)$

7. $u(0,x) = e^{-4x}$

$u(0,x) = f(x) = e^{-4x}$

$u(t,x) = e^{-2t}e^{-4(x-3t)} = e^{-2t-4x+12t}$

$= e^{-4x+10t} = e^{-4x}e^{10t}$

8. $u(0,x) = (1+x^2)^{-1}$

$u(0,x) = f(x) = (1+x^2)^{-1}$

$u(t,x) = e^{-2t}(1+(x-3t)^2)^{-1}$