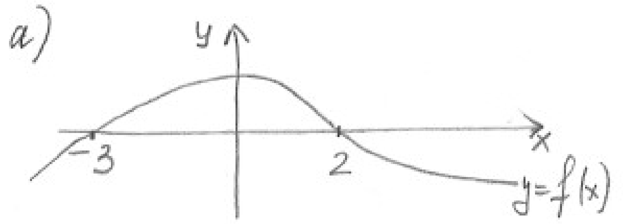
Math 19

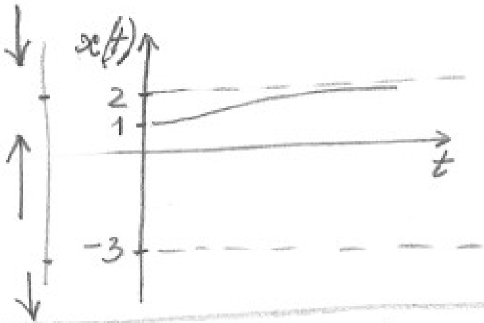
Problem Set #3

Ex. 1, 2, 4, 6 page 71, 72

(1) $\frac{dx}{dt} = f(x)$. Describe what happens to $x(t)$ as t gets large if $x(0) = 1$.

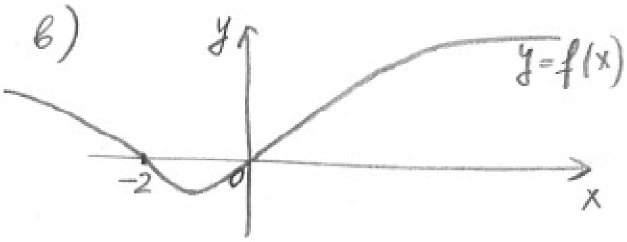
a)

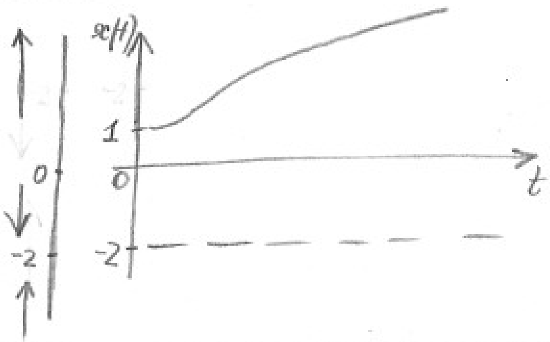




As $t \to \infty$, $x(t) \to 2$;

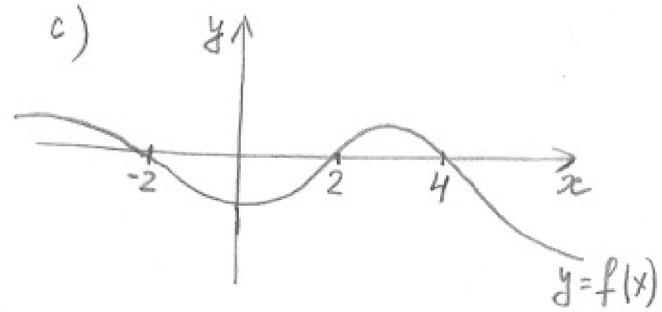
b)

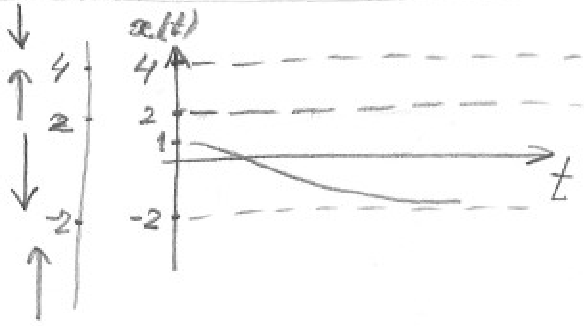




As $t \to \infty$, $x(t) \to \infty$

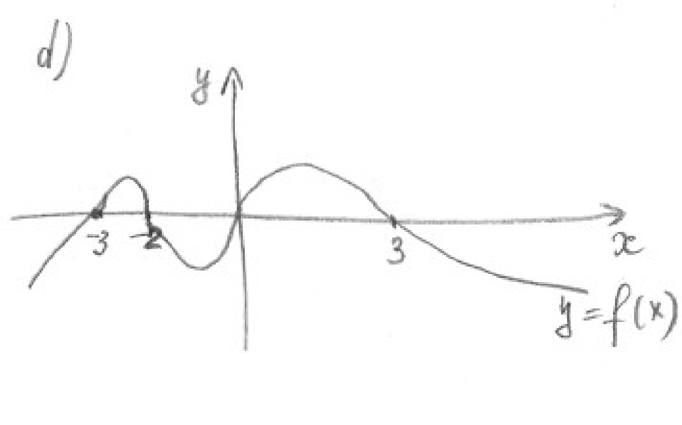
c)

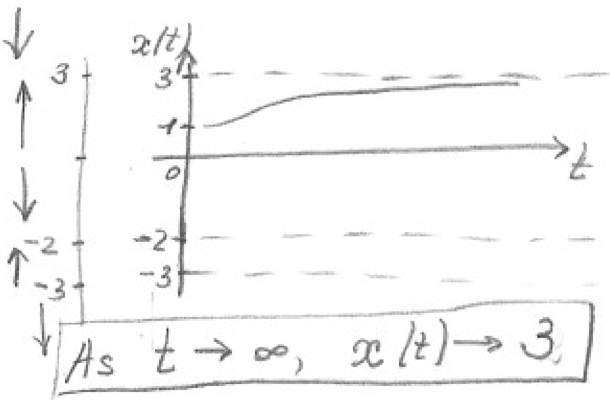




As $t \to \infty$, $x(t) \to -2$

d)

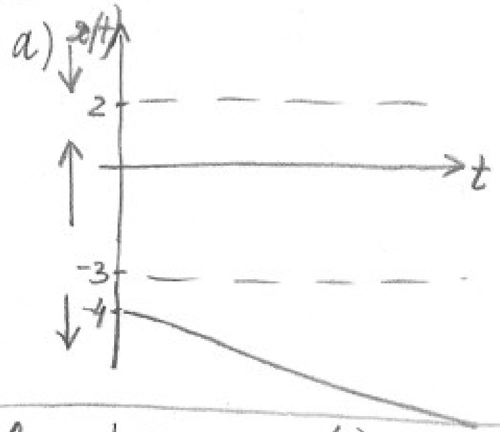




As $t \to \infty$, $x(t) \to 3$

(2) $x(0) = -4$

a)

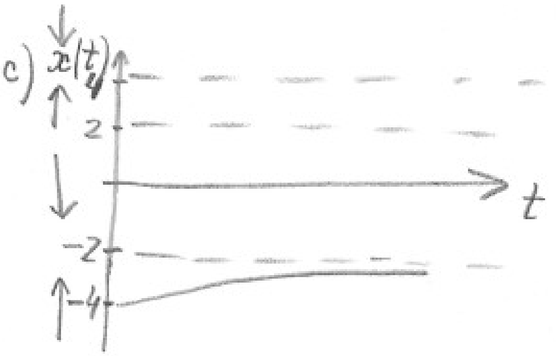


$\boxed{\text{As } t \to \infty,\ x(t) \to -\infty}$

b)

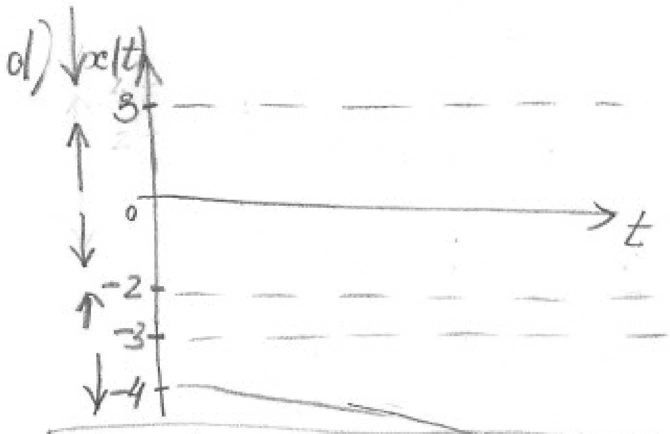
$\boxed{\text{As } t \to \infty,\ x(t) \to -2}$

c)



$\boxed{\text{As } t \to \infty;\ x(t) \to -2}$

d)



$\boxed{\text{As } t \to \infty,\ x(t) \to -\infty}$

(3)

	Stable equilib. points	Unstable equilib. points
a	2	−3
b	−2	0
c	4, −2	2
d	3, −2	0, −3

⑥ $\frac{dn}{dt} = \alpha n - \nu n^2$, α, ν - constants. α, ν - ?

The given equation reminds us of the logistic equation. It involves the carrying capacity.

$$\boxed{\frac{dP}{dt} = k\left(1-\frac{P}{n}\right)P \quad n \text{ - carrying capacity}}$$

$$\frac{dP}{dt} = kP - \frac{k}{n}P^2$$

Given the similarity between the logistic eq. and the one given in the problem, we can write:

$\alpha = k$; $\nu = \frac{k}{n}$ (Notice than n here is the carrying capacity, and n in the original eq $\frac{dn}{dt} = \alpha n - \nu n^2$ is representing the population).

— when the population is low, we have $\left(1-\frac{P}{n}\right) \approx 1 \Rightarrow$
$\Rightarrow \frac{dP}{dt} \approx kP$ which is an exponential growth equation. If we count the population, when it is small, two times, we can find k.

$\frac{dP}{dt} \approx kP \Rightarrow P(t) = P(t_0) + e^{k(t-t_0)}$ By making those 2 counts, we calculate $P(t)$, $P(t_0)$, we know the value of t and t_0 and finding k is just a matter of algebraic calculation. As we've established before $\alpha = k$. We, thus, found α.

— When the population gets larger, it will tend towards the carrying capacity n. When the population reaches an equilibrium $\frac{dP}{dt} = 0$, $\left(1-\frac{P}{n}\right) = 0$, so $\frac{P}{n} = 1$; $P = n$. We can find n this way. We already have the value of k. $\nu = \frac{k}{n}$. This way, we can find ν.