

Problem Set #7  
 Chapter 10, Ex. 1,3,4, p.172;  
 Chapter 11: Ex. 1,2,3,4, p.177

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**Ex. 1, p. 172**

$$\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - x^2 - 2xy \\ 2y - y^2 - 3xy \end{pmatrix}$$

Equilibrium points are:  $(0,0)$ ,  $(1,0)$ ,  $(0,2)$ ,  $(\frac{0.6}{0.2})$ .

$$\mathcal{D} = \begin{pmatrix} 1 - 2x - 2y & -2x \\ -3y & 2 - 2y - 3x \end{pmatrix}$$

Equilib.point	Matrix	$Det(\mathcal{D})$	$Tr(\mathcal{D})$	Stability
$(0,0)$	$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$	$\begin{matrix} 2 \\ (2 > 0) \end{matrix}$	$\begin{matrix} 3 \\ (3 > 0) \end{matrix}$	Unstable
$(0,2)$	$\begin{pmatrix} -3 & 0 \\ -6 & -2 \end{pmatrix}$	$\begin{matrix} 6 \\ (6 > 0) \end{matrix}$	$\begin{matrix} -5 \\ (-5 < 0) \end{matrix}$	Stable
$(1,0)$	$\begin{pmatrix} -1 & -2 \\ 0 & -1 \end{pmatrix}$	$\begin{matrix} 1 \\ (1 > 0) \end{matrix}$	$\begin{matrix} -2 \\ (-2 < 0) \end{matrix}$	Stable
$(\frac{0.6}{0.2})$	$\begin{pmatrix} -0.6 & -1.2 \\ -0.6 & -0.2 \end{pmatrix}$	$\begin{matrix} -0.6 \\ (-0.6 < 0) \end{matrix}$	$\begin{matrix} -0.8 \\ (-0.8 < 0) \end{matrix}$	Unstable

**Ex. 3, p.172**

- $h = x^2y^3$
- $\frac{\partial h}{\partial x} = 2y^3x$
- $\frac{\partial h}{\partial y} = 3x^2y^2$
- $\frac{\partial^2 h}{\partial x^2} = 2y^3$
- $\frac{\partial^2 h}{\partial y^2} = 6x^2y$
- $\frac{\partial^2 h}{\partial y \partial x} = 6xy^2$

$$\frac{\partial^2 h}{\partial x \partial y} = 6xy^2$$

The last two are equal.

- $h = x \cos(xy)$

$$\frac{\partial h}{\partial x} = -xy \sin(xy) + \cos(xy)$$

$$\frac{\partial h}{\partial y} = -x^2 \sin(xy)$$

$$\frac{\partial^2 h}{\partial x^2} = -xy^2 \cos(xy) - 2y \sin(xy)$$

$$\frac{\partial^2 h}{\partial y^2} = -x^3 \cos(xy)$$

$$\frac{\partial^2 h}{\partial y \partial x} = -2x \sin(xy) - x^2 y \cos(xy)$$

$$\frac{\partial^2 h}{\partial x \partial y} = -2x \sin(xy) - x^2 y \cos(xy)$$

The last two are equal.

- $h = \sin(x + y^2)$

$$\frac{\partial h}{\partial x} = \cos(x + y^2)$$

$$\frac{\partial h}{\partial y} = 2y \cos(x + y^2)$$

$$\frac{\partial^2 h}{\partial x^2} = -\sin(x + y^2)$$

$$\frac{\partial^2 h}{\partial y^2} = 2\cos(x + y^2) - 4y^2 \sin(x + y^2)$$

$$\frac{\partial^2 h}{\partial y \partial x} = -2y \sin(x + y^2)$$

$$\frac{\partial^2 h}{\partial x \partial y} = -2y \sin(x + y^2)$$

The last two are equal.

- $h = xe^y$

$$\frac{\partial h}{\partial x} = e^y$$

$$\frac{\partial h}{\partial y} = xe^y$$

$$\frac{\partial^2 h}{\partial x^2} = 0$$

$$\frac{\partial^2 h}{\partial y^2} = xe^y$$

$$\frac{\partial^2 h}{\partial y \partial x} = e^y$$

$$\frac{\partial^2 h}{\partial x \partial y} = e^y$$

The last two are equal.

Thus, we have seen through these examples that  $\frac{\partial^2 h}{\partial y \partial x} = \frac{\partial^2 h}{\partial x \partial y}$ .

**Ex. 4, p. 172**

Integrate  $h(x, y)$  over the indicated rectangle.

a)

$$\int_{-1}^2 \int_0^1 dx dy = \int_{-1}^2 x|_0^1 dy = y|_{-1}^2 = 3$$

b)

$$\int_0^1 \int_{-1}^1 x dx dy = \int_0^1 (x^2/2)|_{-1}^1 dy = 0$$

c)

$$\int_0^1 \int_0^1 (x+y) dx dy = \int_0^1 (x^2/2+yx)|_0^1 dy = \int_0^1 (1/2+y) dy = (y/2+y^2/2)|_0^1 = 1/2+1/2 = 1$$

d)

$$\int_2^3 \int_{-2}^1 xy dx dy = \int_2^3 (x^2y/2)|_{-2}^1 dy = \int_2^3 \frac{y}{2}(1-4) dy = \int_2^3 -\frac{3}{2}y dy = (-3y^2/4)|_2^3 = -15/4$$

e)

$$\int_0^1 \int_0^1 \cos(xy) dx dy = \int_0^1 \frac{\sin(xy)}{y} \Big|_0^1 dy = \int_0^1 \frac{\sin y}{y} dy = 0.946$$

**Chapter 11. Ex. 1, p.177**

a)  $M = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$\vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$M \vec{v} = \begin{pmatrix} 1 \cdot 1 + 2 \cdot 1 \\ 0 \cdot 1 + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

b)  $M = \begin{pmatrix} 3 & 2 \\ -5 & 4 \end{pmatrix}$

$\vec{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$M \vec{v} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$

c)  $M = \begin{pmatrix} 0.5 & 0.2 \\ 8 & 0.4 \end{pmatrix}$

$\vec{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$M \vec{v} = \begin{pmatrix} 0.2 \\ 0.4 \end{pmatrix}$

**Ex. 2, p.177**

	$a$	$b$	$c$
$MM'$	$\begin{pmatrix} 4 & 17 \\ 1 & 6 \end{pmatrix}$	$\begin{pmatrix} 9 & 9 \\ 2 & 11 \end{pmatrix}$	$\begin{pmatrix} 15 & -2 \\ 18 & -15 \end{pmatrix}$
$MM'$	$\begin{pmatrix} 2 & 9 \\ 1 & 8 \end{pmatrix}$	$\begin{pmatrix} 15 & -2 \\ 3 & 5 \end{pmatrix}$	$\begin{pmatrix} 9 & 54 \\ 2 & -9 \end{pmatrix}$

**Ex. 3, p. 177**

	$a$	$b$	$c$
$\det(M)$	1	-27	7
$\det(M)$	2	-4	8

**Ex. 4, p. 177**

Verify that  $\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  is an eigenvector for  $M = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ .

$$\begin{aligned}
M \vec{v} &= \lambda \vec{v} \\
\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} &= \lambda \begin{pmatrix} x \\ y \end{pmatrix} \\
\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{0} \\
\begin{pmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{0} \\
\begin{cases} (1-\lambda)x + 2y = 0 \\ (1-\lambda)y = 0 \end{cases} & \\
\begin{pmatrix} x \\ y \end{pmatrix} &\neq \vec{0}
\end{aligned}$$

Thus,  $1 - \lambda = 0 \Rightarrow \lambda = 1$ . The eigenvalue is  $\lambda = 1$ . Verify that  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  is an eigenvector:

$$\begin{aligned}
M \begin{pmatrix} 1 \\ 0 \end{pmatrix} &= \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} &= \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\end{aligned}$$

Thus,  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  is an eigenvector for  $M$ .