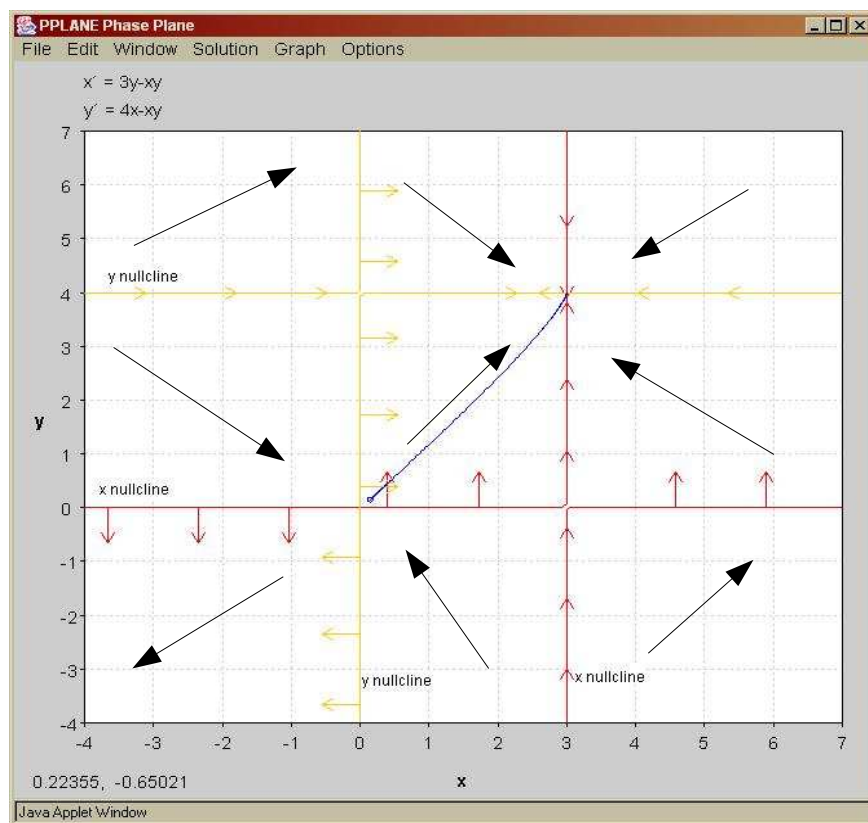


Problem Set #5
Chapter7, Part1: Ex. 1; Part2: Ex. 1-6;

October 22, 2004

Ex. 1, page 125



A trajectory starting at the point $(\frac{1}{10}, \frac{1}{10})$ goes toward the equilibrium point $(3, 4)$. This equilibrium point is a nodal sink, a stable equilibrium point, thus as $t \rightarrow \infty$, the trajectory gets arbitrarily close to this equilibrium point.

Ex.1, page 126.

Compute $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$ when \mathbf{v} and \mathbf{w} equal:

a) $\mathbf{v} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$$\mathbf{v} + \mathbf{w} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$\mathbf{v} - \mathbf{w} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$$

b) $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -10 \\ -31 \end{pmatrix}$

$$\mathbf{v} + \mathbf{w} = \begin{pmatrix} -7 \\ -30 \end{pmatrix}$$

$$\mathbf{v} - \mathbf{w} = \begin{pmatrix} 13 \\ 32 \end{pmatrix}$$

c) $\mathbf{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$

$$\mathbf{v} + \mathbf{w} = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$$

$$\mathbf{v} - \mathbf{w} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

d) $\mathbf{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$$\mathbf{v} + \mathbf{w} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\mathbf{v} - \mathbf{w} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Ex. 2, page 168.

Compute $r\mathbf{v}$ when r and \mathbf{v} are given

a) $r = 3$ and $\mathbf{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$$r\mathbf{v} = \begin{pmatrix} -9 \\ 6 \end{pmatrix}$$

b) $r = 1$ and $\mathbf{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$$r\mathbf{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

c) $r = 0$ and $\mathbf{v} = \begin{pmatrix} -10 \\ -31 \end{pmatrix}$

$$r\mathbf{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

d) $r = -2$ and $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$$r\mathbf{v} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$$

Ex. 3, page 126

Compute the length of the vectors \mathbf{v} in Exercise 2.

In order to compute the length of these two vectors, we can use the dot product. We know that $|\mathbf{v}| = \sqrt{\mathbf{v} \cdot \mathbf{v}}$.

a) $\mathbf{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$$|\mathbf{v}| = \sqrt{\begin{pmatrix} -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \end{pmatrix}} = \sqrt{13}$$

b) $\mathbf{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$$|\mathbf{v}| = \sqrt{\begin{pmatrix} 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \end{pmatrix}} = \sqrt{4} = 2$$

c) $\mathbf{v} = \begin{pmatrix} -10 \\ -31 \end{pmatrix}$

$$|\mathbf{v}| = \sqrt{\begin{pmatrix} -10 \\ -31 \end{pmatrix} \cdot \begin{pmatrix} -10 \\ -31 \end{pmatrix}} = \sqrt{1061}$$

d) $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$$|\mathbf{v}| = \sqrt{\begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}} = \sqrt{10}$$

Note: We can also find the length of these vectors using the Pythagorean Theorem.

Ex. 4, page 126

Compute $\mathbf{v} \cdot \mathbf{w}$ for the vectors \mathbf{v} and \mathbf{w} , in Exercise 1.

a) $\mathbf{v} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$$\mathbf{v} \cdot \mathbf{w} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \end{pmatrix} = 5 \cdot (-3) + 5 \cdot 2 = -5$$

b) $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -10 \\ -31 \end{pmatrix}$

$$\mathbf{v} \cdot \mathbf{w} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -10 \\ -31 \end{pmatrix} = 3 \cdot (-10) + 1 \cdot (-31) = -61$$

c) $\mathbf{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$

$$\mathbf{v} \cdot \mathbf{w} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \end{pmatrix} = 2 \cdot 6 + 0 \cdot (-1) = 12$$

d) $\mathbf{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$$\mathbf{v} \cdot \mathbf{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} = 0 \cdot (-1) + 1 \cdot 0 = 0$$

Ex.5, page 126.

If $\mathbf{v}(t)$ is the vector function of time given, compute $\frac{d}{dt}\mathbf{v}$:

a) $\mathbf{v} = \begin{pmatrix} \cos(2t) \\ 3e^{-2t} \end{pmatrix}$.
 $\mathbf{v}' = \begin{pmatrix} (\cos(2t))' \\ (3e^{-2t})' \end{pmatrix} = \begin{pmatrix} -2\sin(2t) \\ -6e^{-2t} \end{pmatrix}$.

b) $\mathbf{v} = \begin{pmatrix} e^{2t} \\ t^2 \end{pmatrix}$.
 $\mathbf{v}' = \begin{pmatrix} (e^{2t})' \\ (t^2)' \end{pmatrix} = \begin{pmatrix} 2e^{2t} \\ 2t \end{pmatrix}$.

c) $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.
 $\mathbf{v}' = \begin{pmatrix} (3)' \\ (1)' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

d) $\mathbf{v} = \begin{pmatrix} -\sin(t) \\ 2t \end{pmatrix}$.
 $\mathbf{v}' = \begin{pmatrix} (-\sin(t))' \\ (2t)' \end{pmatrix} = \begin{pmatrix} -\cos(t) \\ 2 \end{pmatrix}$.

Ex.6, page 126.

Compute the antiderivate for the vectors \mathbf{v} in Exercise 5.

a) $\mathbf{v} = \begin{pmatrix} \cos(2t) \\ 3e^{-2t} \end{pmatrix}$.
 $\int \mathbf{v} dt = \begin{pmatrix} \int \cos(2t) dt \\ \int 3e^{-2t} dt \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\sin(2t)+C \\ -\frac{3}{2}e^{-2t}+C \end{pmatrix}$.

b) $\mathbf{v} = \begin{pmatrix} e^{2t} \\ t^2 \end{pmatrix}$.
 $\int \mathbf{v} dt = \begin{pmatrix} \int e^{2t} dt \\ \int t^2 dt \end{pmatrix} = \begin{pmatrix} \frac{1}{2}e^{2t}+C \\ \frac{1}{3}t^3+C \end{pmatrix}$.

c) $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.
 $\int \mathbf{v} dt = \begin{pmatrix} \int 3 dt \\ \int 1 dt \end{pmatrix} = \begin{pmatrix} 3t+C \\ t+C \end{pmatrix}$.

d) $\mathbf{v} = \begin{pmatrix} -\sin(t) \\ 2t \end{pmatrix}$.
 $\int \mathbf{v} dt = \begin{pmatrix} \int -\sin(t) dt \\ \int 2t dt \end{pmatrix} = \begin{pmatrix} \cos(t)+C \\ t^2+C \end{pmatrix}$.