Problem Set #5
Chapter7, Part1: Ex. 1; Part2: Ex. 1-6;

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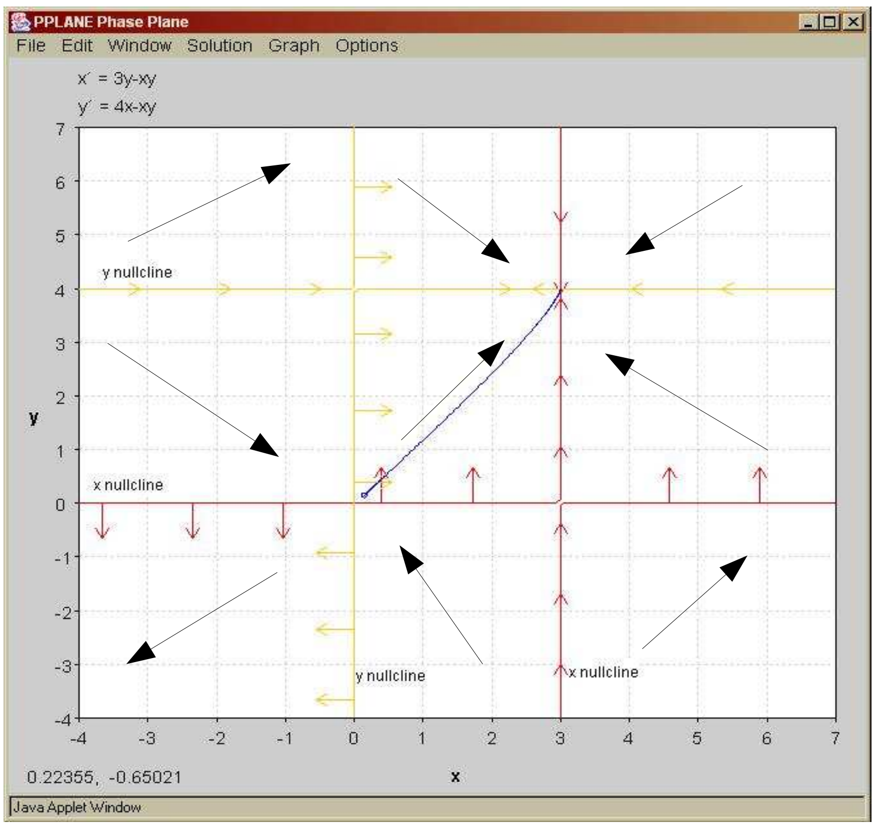
Ex. 1, page 125



A trajectory starting at the point $(\frac{1}{10}, \frac{1}{10})$ goes toward the equilibrium point $(3, 4)$. This equilibrium point is a nodal sink, a stable equilibrium point, thus as $t \to \infty$, the trajectory gets arbitrarily close to this equilibrium point.

Ex.1, page 126.

Compute $\mathbf{v}+\mathbf{w}$ and $\mathbf{v}-\mathbf{w}$ when $\mathbf{v}$ and $\mathbf{w}$ equal:

a) $\mathbf{v} = \binom{5}{5}$ and $\mathbf{w} = \binom{-3}{2}$

$\mathbf{v}+\mathbf{w} = \binom{2}{7}$

$\mathbf{v}-\mathbf{w} = \binom{8}{3}$

b) $\mathbf{v} = \binom{3}{1}$ and $\mathbf{w} = \binom{-10}{-31}$

$\mathbf{v}+\mathbf{w} = \binom{-7}{-30}$

$\mathbf{v}-\mathbf{w} = \binom{13}{32}$

c) $\mathbf{v} = \binom{2}{0}$ and $\mathbf{w} = \binom{6}{-1}$

$\mathbf{v}+\mathbf{w} = \binom{8}{-1}$

$\mathbf{v}-\mathbf{w} = \binom{-4}{1}$

d) $\mathbf{v} = \binom{0}{1}$ and $\mathbf{w} = \binom{-1}{0}$

$\mathbf{v}+\mathbf{w} = \binom{-1}{1}$

$\mathbf{v}-\mathbf{w} = \binom{1}{1}$

Ex. 2, page 168.

Compute $r\mathbf{v}$ when r and $\mathbf{v}$ are given

a) $r = 3$ and $\mathbf{v} = \binom{-3}{2}$

$r\mathbf{v} = \binom{-9}{6}$

b) $r = 1$ and $\mathbf{v} = \binom{2}{0}$

$r\mathbf{v} = \binom{2}{0}$

c) $r = 0$ and $\mathbf{v} = \binom{-10}{-31}$

$r\mathbf{v} = \binom{0}{0}$

d) $r = -2$ and $\mathbf{v} = \binom{3}{1}$

$r\mathbf{v} = \binom{-6}{-2}$

Ex. 3, page 126

Compute the length of the vectors $\mathbf{v}$ in Exercise 2.

In order to compute the length of these two vectors, we can use the dot product. We know that $|\mathbf{v}| = \sqrt{\mathbf{v} \cdot \mathbf{v}}$.

a) $\mathbf{v} = \binom{-3}{2}$

$|\mathbf{v}| = \sqrt{\binom{-3}{2} \cdot \binom{-3}{2}} = \sqrt{13}$

b) $\mathbf{v} = \binom{2}{0}$

$|\mathbf{v}| = \sqrt{\binom{2}{0} \cdot \binom{2}{0}} = \sqrt{4} = 2$

c) $\mathbf{v} = \binom{-10}{-31}$

$|\mathbf{v}| = \sqrt{\binom{-10}{-31} \cdot \binom{-10}{-31}} = \sqrt{1061}$

d) $\mathbf{v} = \binom{3}{1}$

$|\mathbf{v}| = \sqrt{\binom{3}{1} \cdot \binom{3}{1}} = \sqrt{10}$

Note: We can also find the length of these vectors using the Pythagorean Theorem.

Ex. 4, page 126

Compute $\mathbf{v} \cdot \mathbf{w}$ for the vectors $\mathbf{v}$ and $\mathbf{w}$, in Exercise 1.

a) $\mathbf{v} = \binom{5}{5}$ and $\mathbf{w} = \binom{-3}{2}$

$\mathbf{v} \cdot \mathbf{w} = \sqrt{\binom{5}{5} \cdot \binom{-3}{2}} = 5 \cdot (-3) + 5 \cdot 2 = -5$

b) $\mathbf{v} = \binom{3}{1}$ and $\mathbf{w} = \binom{-10}{-31}$

$\mathbf{v} \cdot \mathbf{w} = \sqrt{\binom{3}{1} \cdot \binom{-10}{-31}} = 3 \cdot (-10) + 1 \cdot -31 = -61$

c) $\mathbf{v} = \binom{2}{0}$ and $\mathbf{w} = \binom{6}{-1}$

$\mathbf{v} \cdot \mathbf{w} = \sqrt{\binom{2}{0} \cdot \binom{6}{-1}} = 2 \cdot 6 + 0 \cdot -1 = 12$

d) $\mathbf{v} = \binom{0}{1}$ and $\mathbf{w} = \binom{-1}{0}$

$\mathbf{v} \cdot \mathbf{w} = \sqrt{\binom{0}{1} \cdot \binom{-1}{0}} = 0 \cdot (-1) + 1 \cdot 0 = 0$

Ex.5, page 126.

If $\mathbf{v}(t)$ is the vector function of time given, compute $\frac{d}{dt}\mathbf{v}$:

a) $\mathbf{v} = \binom{\cos(2t)}{3e^{-2t}}$.

$\mathbf{v}' = \binom{(\cos(2t))'}{(3e^{-2t})'} = \binom{-2\sin(2t)}{-6e^{-2t}}$.

b) $\mathbf{v} = \binom{e^{2t}}{t^2}$.

$\mathbf{v}' = \binom{(e^{2t})'}{(t^2)'} = \binom{2e^{2t}}{2t}$.

c) $\mathbf{v} = \binom{3}{1}$.

$\mathbf{v}' = \binom{(3)'}{(1)'} = \binom{0}{0}$.

d) $\mathbf{v} = \binom{-\sin(t)}{2t}$.

$\mathbf{v}' = \binom{(-\sin(t))'}{(2t)'} = \binom{-\cos(t)}{2}$.

Ex.6, page 126.

Compute the antiderivate for the vectors $\mathbf{v}$ in Exercise 5.

a) $\mathbf{v} = \binom{\cos(2t)}{3e^{-2t}}$.

$\int \mathbf{v}dt = \binom{\int \cos(2t)dt}{\int 3e^{-2t}dt} = \binom{\frac{1}{2}\sin(2t)+C}{-\frac{3}{2}e^{-2t}+C}$.

b) $\mathbf{v} = \binom{e^{2t}}{t^2}$.

$\int \mathbf{v}dt = \binom{\int e^{2t}dt}{\int t^2dt} = \binom{\frac{1}{2}e^{2t}+C}{\frac{1}{3}t^3+C}$.

c) $\mathbf{v} = \binom{3}{1}$.

$\int \mathbf{v}dt = \binom{\int 3dt}{\int 1dt} = \binom{3t+C}{t+C}$.

d) $\mathbf{v} = \binom{-\sin(t)}{2t}$.

$\int \mathbf{v}dt = \binom{\int -\sin(t)dt}{\int 2tdt} = \binom{\cos(t)+C}{t^2+C}$.