Math 19. Lecture 5
Stability in a One-Component System

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1 The North Atlantic Fishery

- What is the history of the New England fishery and what are the issues?

- What about the fishery in the northern Pacific? What are the issues?

2 Initial Values of $x(t)$

- The general solution to
$$\frac{dx}{dt} = ax$$
is
$$x(t) = x(0)e^{at}.$$
We sometimes use the word *trajectory* for the function $x(t)$.

- The general solution to
$$\frac{dx}{dt} = ax^n$$
is
$$x(t) = x(0)\left(1 - a(n-1)x(0)^{n-1}t\right)^{-1/(n-1)}.$$

3 Solutions—Existence and Uniqueness

An equation of the form
$$\frac{dx}{dt} = f(x)$$

for a function x of the variable t is predictive in the sense that if you choose any value of x to start, then there is precisely one solution to this equation that starts at your chosen value.

4   Stability

- An *equilibrium solution* is constant for all values of the independent variable. The graph is a horizontal line. Equilibrium values can be identified by setting the derivative of the function equal to zero.

- An equilibrium solution is *stable* if a small change in the initial conditions gives a solution which tends toward the equilibrium as the independent variable tends towards positive infinity.

- An equilibrium solution is *unstable* if a small change in the initial conditions gives a solution which veers away from the equilibrium as the independent variable tends towards positive infinity.

5   What Happens in Nature?

Unstable equilibrium points are rarely seen in natural systems.

Homework

- Chapter 4. Exercises 1, 2, 4, 6; pp. 71–72.

Readings and References

- C. Taubes. *Modeling Differential Equations in Biology.* Prentice Hall, Upper Saddle River, NJ, 2001. Chapter 4.

- "Biologists Sort the Lessons of Fisheries Collapse," pp. 53–55.

- "New Study Provides some Good News for Fisheries," pp. 55–57.

- "Population Dynamics of Exploited Fish Stocks at Low Population Levels," pp. 57–62.