Math 19. Mathematical Modeling
Review Guide for Midterm II

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The second midterm will primarily cover Chapters 13–24, but you will also be responsible for any material from the first part of the course that is needed in these chapters. The exam will be in Thursday, December 15 at 7-9 PM in Science A. You may bring one 4×6 index card of notes. Calculators will be permitted, but most of the problems should only require a minimum of numerical computation. There will be a review session on Monday, December 12 at 7-9 PM in Science A.

- Go over the extra problems and solutions for Chapters 13–25 (pp. 489–497).
- You should understand and be able to apply the advection equation (pp. 192–197),
$$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x} + f(u).$$
- You should understand and be able to apply the diffusion equation (pp. 216–219),
$$\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial x^2} + f(u).$$
- You should understand and be able to apply Laplace's equation (pp. 260–263),
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

- You should understand and be able to apply initial conditions and boundary conditions to the advection equation, the diffusion equation, and Laplace's equation (pp. 246–254).

- You should understand and be able to apply the separation of variables technique to solve advection equation, the diffusion equation, and Laplace's equation (pp. 246–254, 259–263).

- You should be able to solve the differential equation

$$\frac{d^2u}{dx^2} = cu$$

  for $c > 0$, $c = 0$, and $c < 0$ (pp. 261–262).

- You should understand and be able to apply the Principle of Superposition (pp. 227–229).

- You should understand and be able to apply stability criterion for equilibrium solutions (pp. 266–273, 311–316).

- You should understand the difference between advection and diffusion. That is, the advection equation is used when particle motion is due the motion of the ambient fluid. On the other hand, the diffusion equation is used when the particles move by random drift (pp. 318–322).

- You should understand how the maximum principle can be used as a tool to verify the stability of an equilibrium solution (pp. 321–322).

- You should understand that reaction-diffusion equations can have solutions that look like traveling waves (pp. 351–357).

- You should understand that the traveling wave form of a solution to the reaction-diffusion equation (pp. 351–357).

- You should understand that the assumption of the solution that is a traveling wave for a reaction-diffusion equation turns the reaction-diffusion equation into and ODE (pp. 351–357).

- You should know how phase plane analysis can be used to analyze the solutions to an ODE (pp. 351–357).

- You should understand how, in some cases, the speed of the traveling wave solution of a reaction-diffusion equation is determined by the wave's shape (pp. 365–368).

- You should understand that a differential equation for two unknown functions of time can have time-periodic solutions (pp. 375–380).

- You should understand that time periodic solutions must occur if there is a basin of attraction that has inside it a single equilibrium point that is unstable and repelling. In particular, this time-periodic solution will move in the basin of attraction and is stable in the sense that nearby trajectories spiral into it (pp. 375–380).

- Some fairly simple differential equations for two unknown functions of time that exhibit cyclic motion that is relatively fast in one part of the cycle and relatively slow in another. Fast/slow dichotomy can arise in a system that has two interacting parts, one always slow moving and the other with the potential for much faster movement. You should understand how the motion of the system as a whole moves slowly as long as the fast subsystem tracks a stable equilibrium point whose position is determined by the slow subsystem. The motion speeds up when the motion of the slow subsystem makes the equilibrium point in the fast subsystem either disappear or become unstable. You should be able to apply the governing equations to estimate the elapsed time in various parts of the cycle (pp. 393–401).