key

Math 19 Problem Set #6: p138-139 Ex. 1-5, 7; p. 145-146 Ex. 1, 2, 4, 5

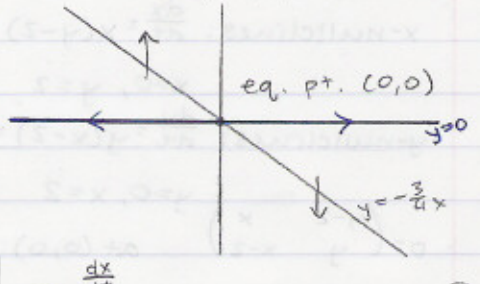
Ch. 8

1. $\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3x+4y\\y\end{pmatrix}$

x-nullcline: $\frac{dx}{dt} = 3x+4y = 0$

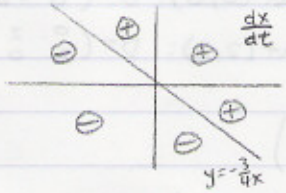
$y = -\frac{3}{4}x$

y-nullcline: $\frac{dy}{dt} = y = 0$



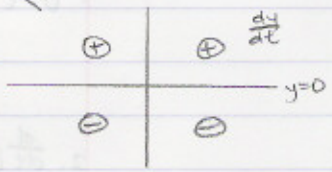
2. $\frac{dx}{dt}(1,0) = 3(1)+4(0) = 3$

$\frac{dx}{dt}(-1,0) = 3(-1)+4(0) = -3$



$\frac{dy}{dt}(4,-3) = -3$

$\frac{dy}{dt}(-4,3) = 3$



3. $\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x-2y\\x+y\end{pmatrix}$

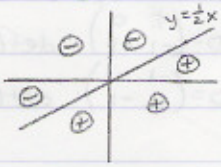
x-nullcline: $\frac{dx}{dt} = x-2y = 0$ $\quad y = \frac{1}{2}x$

y-nullcline: $\frac{dy}{dt} = x+y = 0$ $\quad y = -x$

eq. pts. = intersection of nullclines = (0,0)

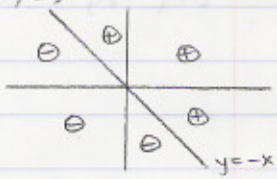
4. $\frac{dx}{dt}(1,-1) = 3$

$\frac{dx}{dt}(-1,1) = -3$



$\frac{dy}{dt}(2,1) = 3$

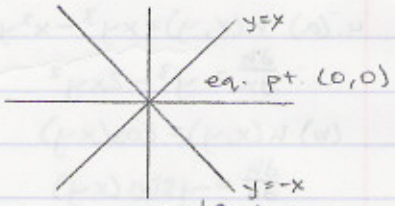
$\frac{dy}{dt}(-2,-1) = -3$



5. $\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-x-y\\x-y\end{pmatrix}$

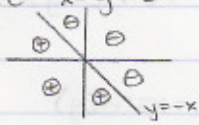
x-nullcline: $\frac{dx}{dt} = -x-y = 0$ $\quad y = -x$

y-nullcline: $\frac{dy}{dt} = x-y = 0$ $\quad y = x$



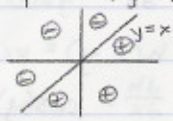
$\frac{dx}{dt}(1,1) = -2$

$\frac{dx}{dt}(-1,-1) = 2$



$\frac{dy}{dt}(1,-1) = 2$

$\frac{dy}{dt}(-1,1) = -2$



7. Let $\frac{dx}{dt} = f(x)$ & $\frac{dy}{dt} = g(x)$. $\quad D = \begin{pmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y}\end{pmatrix}$ stable if $\det(D)>0$ & $\mathrm{tr}(D)<0$

(1) $\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3x+4y\\y\end{pmatrix}$ $\quad D = \begin{pmatrix}3 & 4\\0 & 1\end{pmatrix}$

$\det(D) = 3(1)-4(0) = 3>0$ $\quad \mathrm{tr}(D) = 3+1 = 4>0$ unstable

(3) $\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x-2y\\x+y\end{pmatrix}$ $\quad D = \begin{pmatrix}1 & -2\\1 & 1\end{pmatrix}$

$\det(D) = 1(1)-1(-2) = 3>0$ $\quad \mathrm{tr}(D) = 1+1 = 2>0$ unstable

(5) $\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-x-y\\x+y\end{pmatrix}$ $\quad D = \begin{pmatrix}-1 & -1\\1 & -1\end{pmatrix}$

$\det(D) = -1(-1)-(-1)(1) = 2>0$ $\quad \mathrm{tr}(D) = -1+(-1) = -2<0$ stable

Ch.9

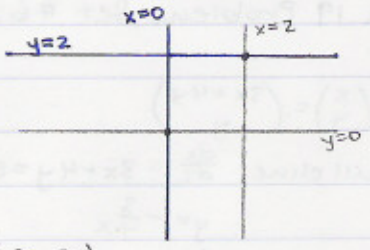
1. $\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}xy-2x\\xy-2y\end{pmatrix}$

x-nullclines: $\frac{dx}{dt}=x(y-2)=0$

$x=0,\ y=2$

y-nullclines: $\frac{dy}{dt}=y(x-2)=0$

$y=0,\ x=2$



Eq. pts.:
(0,0)
(2,2)

$D=\begin{pmatrix}y-2 & x\\ y & x-2\end{pmatrix}$

at (0,0): $D=\begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}$ $\det(D)=4$ $\mathrm{tr}(D)=-4$ stable

at (2,2): $D=\begin{pmatrix}0 & 2\\ 2 & 0\end{pmatrix}$ $\det(D)=-4$ $\mathrm{tr}(D)=0$ unstable

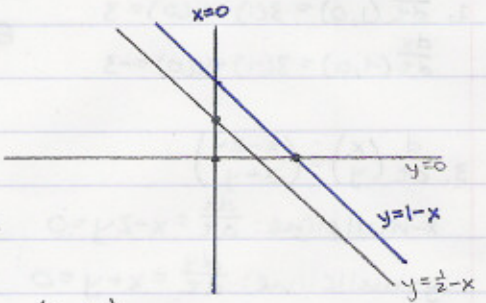
2. $\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x-x^2-xy\\y-2xy-2y^2\end{pmatrix}$

x-nullclines: $\frac{dx}{dt}=x(1-x-y)=0$

$x=0,\ y=1-x$

y-nullclines: $\frac{dy}{dt}=y(1-2x-2y)=0$

$y=0,\ y=\frac{1}{2}-x$



Eq. pts:
(0,0)
$(0,\frac{1}{2})$
(1,0)

$D=\begin{pmatrix}1-2x-y & -x\\ -2y & 1-2x-4y\end{pmatrix}$

at (0,0): $D=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $\det(D)=1$ $\mathrm{tr}(D)=2$ unstable

at $(0,\frac{1}{2})$: $D=\begin{pmatrix}\frac{1}{2} & 0\\ -1 & -1\end{pmatrix}$ $\det(D)=-\frac{1}{2}$ $\mathrm{tr}(D)=-\frac{3}{2}$ unstable

at (1,0): $D=\begin{pmatrix}-1 & -1\\ 0 & -1\end{pmatrix}$ $\det(D)=1$ $\mathrm{tr}(D)=-2$ stable

4. (a) $h(x,y)=xy^3-x^2y^2$

$\frac{dh}{dx}=y^3-2xy^2$ $\quad\frac{dh}{dy}=3xy^2-2x^2y$

(b) $h(x,y)=\cos(xy)$

$\frac{dh}{dx}=-y\sin(xy)$ $\quad\frac{dh}{dy}=-x\sin(xy)$

(c) $h(x,y)=x\cos(y)-1$

$\frac{dh}{dx}=\cos(y)$ $\quad\frac{dh}{dy}=-x\sin(y)$

(d) $h(x,y)=y\cos(x)-x^2$

$\frac{dh}{dx}=-y\sin(x)-2x$ $\quad\frac{dh}{dy}=\cos(x)$

5. $x(t)=\cos t \qquad y(t)=\sin t$

(a) a: $h(x(t),y(t))=\cos t\sin^3 t-\cos^2 t\sin^2 t$

b: $h(x(t),y(t))=\cos(\cos t\sin t)$

c: $h(x(t),y(t))=\cos t\cos(\sin t)-1$

d: $h(x(t),y(t))=\sin t\cos(\cos t)-\cos^2 t$

(b) a: $\frac{dh}{dt} = -\sin^4 t + 3\sin^2 t\cos^2 t + 2\cos t\sin^3 t - 2\sin t\cos^3 t$

b: $\frac{dh}{dt} = -\sin(\cos t\sin t)(-\sin^2 t + \cos^2 t)$

c: $\frac{dh}{dt} = -\sin t\cos(\sin t) - \sin(\sin t)\cos^2 t$

d: $\frac{dh}{dt} = \cos t\cos(\cos t) + \sin(\cos t)\sin^2 t + 2\cos t\sin t$

(c) $\frac{dh}{dt} = \frac{dh}{dx}\cdot\frac{dx}{dt} + \frac{dh}{dy}\cdot\frac{dy}{dt}$. $\quad \frac{dx}{dt} = -\sin t \quad \frac{dy}{dt} = \cos t$

a: $\frac{dh}{dt} = (y^3 - 2xy^2)(-\sin t) + (3xy^2 - 2x^2 y)\cos t$

$= (\sin^3 t - 2\cos t\sin^2 t)(-\sin t) + (3\cos t\sin^2 t - 2\cos^2 t\sin t)\cos t$

b: $\frac{dh}{dt} = (-y\sin(xy))(-\sin t) + (-x\sin(xy))\cos t$

$= (-\sin t\sin(\cos t\sin t))(-\sin t) + (-\cos t\sin(\cos t\sin t))\cos t$

c: $\frac{dh}{dt} = \cos y(-\sin t) + (-x\sin(y))\cos t$

$= \cos(\sin t)(-\sin t) + (-\cos t\sin(\sin t))\cos t$

d: $\frac{dh}{dt} = (-y\sin x - 2x)(-\sin t) + \cos x\cos t$

$= (-\sin t\sin(\cos t) - 2\cos t)(-\sin t) + \cos(\cos t)\cos t$