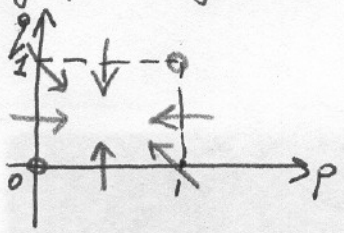
HW #18 Math 19

Chapter 23; Ex #1 (a, c, e), 2 (a, c)

① Square $\{(p,q): 0 \le p \le 1;\ 0 \le q \le 1\}$ $\quad \frac{d}{ds}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} f(p,q) \\ g(p,q) \end{pmatrix}$

For which of the following f and g is this square a basin of attraction?

a) $f(p,q) = q - p$
$g(p,q) = p - q$



On the sides:

$p = 1 \quad \frac{dp}{ds} = q - p = q - 1;\ 0 \le q \le 1 \Rightarrow \frac{dp}{ds} \le 0$

$p = 0 \quad \frac{dp}{ds} = q; \quad 0 \le q \le 1 \Rightarrow \frac{dp}{ds} \ge 0$

$q = 0 \quad \frac{dq}{ds} = p - q = p \quad 0 \le p \le 1 \Rightarrow \frac{dq}{ds} \ge 0$

$q = 1 \quad \frac{dq}{ds} = p - q \quad 0 \le p \le 1 \Rightarrow \frac{dq}{ds} \le 0$

At the corners:

$(0,0) \quad \frac{dp}{ds} = 0 \quad \frac{dq}{ds} = 0$

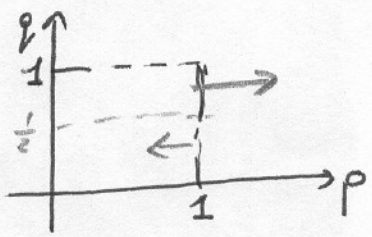
$(0,1) \quad \frac{dp}{ds} = 1 \quad \frac{dq}{ds} = -1$

$(1,0) \quad \frac{dp}{ds} = -1 \quad \frac{dq}{ds} = 1$

$(1,1) \quad \frac{dp}{ds} = 0 \quad \frac{dq}{ds} = 0$

Yes, it is a basin of attraction

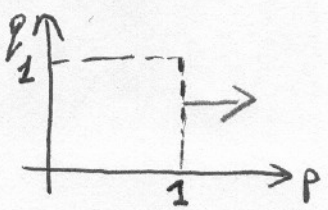
c) $f(p,q) = p(q - \frac{1}{2})$
$g(p,q) = -q$



$p = 1 \quad \frac{dp}{ds} = p(q - \frac{1}{2})$

Not a basin of attraction because when $q > \frac{1}{2}$ and $p = 1 \Rightarrow \frac{dp}{ds} > 0$;

e) $f(p,q) = qp(p - \frac{1}{2})$
$g(p,q) = pq - 1$



$p = 1 \quad \frac{dp}{ds} = \frac{q}{2}; \quad 0 \le q \le 1 \Rightarrow \frac{dp}{ds} > 0.$ Thus not a basin of attraction.

(2) a) $\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ $\quad Tr = 4 > 0$, $\det = 3 - 2 = 1 > 0$ $\Rightarrow$ Repelling

c) $\begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix}$ $\quad Tr = -2 < 0$, $\det = -3 + 4 = 1 > 0$ $\Rightarrow$ Stable