HW #16 Math 19

Chapter 21. Ex. 1, 2 (a, c), 3 page 364

① $\frac{\partial u}{\partial t} = \frac{\partial f}{\partial t} = \frac{df}{ds} \cdot \frac{\partial s}{\partial t} = -c\,\frac{df}{ds}$

* We assume $u(t,x) = f(x-ct)$

$\frac{\partial u}{\partial x} = \frac{\partial f}{\partial x} = \frac{df}{ds} \cdot \frac{\partial s}{\partial x} = \frac{df}{ds}$

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 f}{\partial x^2} = \frac{d}{ds}\left(\frac{\partial f}{\partial x}\right)\frac{\partial s}{\partial x} = \frac{d^2 f}{ds^2}$

Now the exercise is reduced to a simple substitution exercise:

a) $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2} + 2(u - \sin(u)) \Rightarrow -c\frac{df}{ds} = 3\frac{d^2 f}{ds^2} + 2(f - \sin(f))$

b) $\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial u}{\partial x} + u^3 \Rightarrow -c\frac{df}{ds} = \mu\frac{d^2 f}{ds^2} - 6\frac{df}{ds} + f^3$

c) $\frac{\partial u}{\partial t} = -5\frac{\partial^2 u}{\partial x^2} + e^u \Rightarrow -c\frac{df}{ds} = -5\frac{d^2 f}{ds^2} + e^f$

[* Note: there was a mistake in the book for #1,c. It should be $\frac{\partial^2 u}{\partial x^2}$.]

d) $\frac{\partial^2 u}{\partial t^2} = \mu\frac{\partial^2 u}{\partial x^2} - 25u^2 \Rightarrow c^2\frac{d^2 f}{ds^2} = \mu\frac{d^2 f}{ds^2} - 25 f^2$

② $\boxed{\text{for } 1a}$ a) $\frac{df}{ds} = p \Rightarrow \frac{d^2 f}{ds^2} = \frac{dp}{ds}$

From #1a, we have: $-cp = 3\frac{dp}{ds} + 2(f - \sin(f)) \Rightarrow$

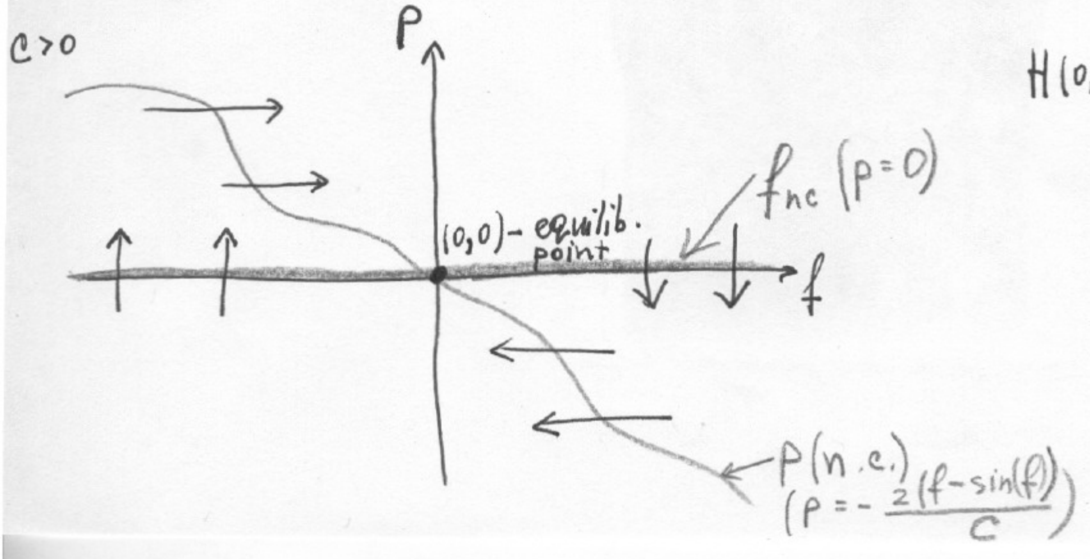
b) $\Rightarrow \begin{cases} \frac{dp}{ds} = \frac{-cp - 2(f - \sin(f))}{3} \\ \frac{df}{ds} = p \end{cases}$

The nullclines are:

$p_{n.c.} \Rightarrow -cp - 2(f - \sin(f)) = 0 \Rightarrow p = -\frac{2(f-\sin(f))}{c}$

$f\ n.c. \Rightarrow p = 0$

$c > 0$



$H(0,0) = \begin{pmatrix} -\frac{c}{3} & -\frac{2}{3} - \frac{2}{3}\cos(0) \\ 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} -\frac{c}{3} & -\frac{4}{3} \\ 1 & 0 \end{pmatrix}$

$\left.\begin{array}{l} tr = -\frac{c}{3} < 0 \\ det = \frac{4}{3} > 0 \end{array}\right\}$ (0,0) is a stable equilibrium point

$\boxed{\text{for } 1\,C}$

a) $\dfrac{df}{ds} = p \Rightarrow \dfrac{d^2 f}{ds^2} = \dfrac{dp}{ds}$

$$\frac{df}{ds} = p$$

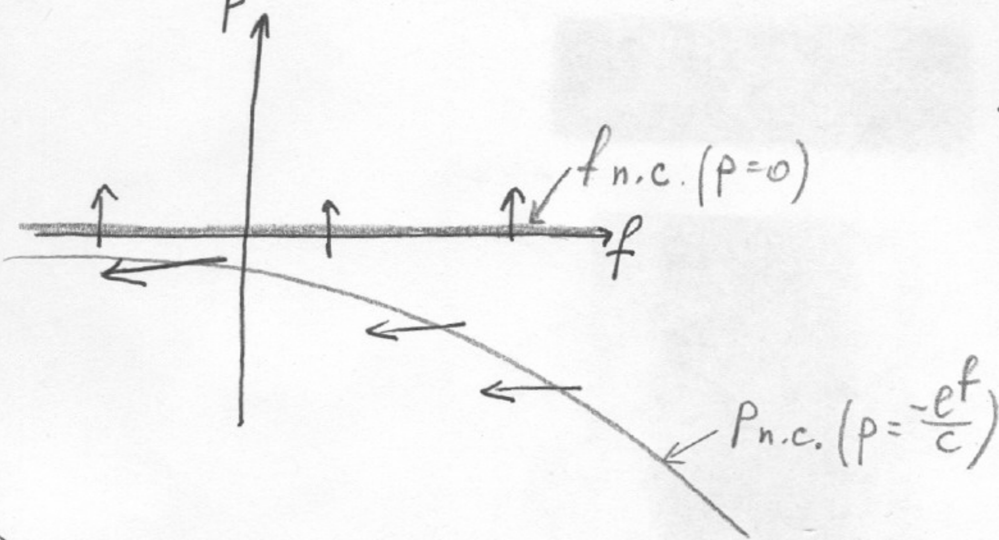
$$\frac{dp}{ds} = \frac{cp + e^f}{5}$$

From #1 c, we have $-cp = -5\dfrac{dp}{ds} + e^f$

The nullclines are:

f n.c. $\Rightarrow p = 0$

p n.c. $\Rightarrow p = \dfrac{-e^f}{c}$



There is no intersection point between the 2 nullclines $\Rightarrow$ there is no equilibrium point.

(3) Does not account for:

Mouse food supply
Weather limitations
Seasonal changes
Mating patterns
Human influence
Development of resistance
Incubation period of the virus