Key

Math 19 Problem Set #15: p 349-350 Ex. 1, 2

1. (a) Advection - venom moves at constant speed in blood (ambient fluid.

(b) Diffusion - mosquitos move randomly

(c) Advection - the Gulf Stream current is the ambient fluid

(d) Diffusion - oil slick spreads randomly because there is no single strong current in the lake.

2. $0 \le x \le 1$

(a) $\lambda g = \frac{d^2g}{dx^2} - (x^2+1)g$ $\quad (x^2+1) > 0$ on $[0,1]$

max: $g > 0$, $\frac{d^2g}{dx^2} < 0$

$\lambda g = \underbrace{\frac{d^2g}{dx^2}}_{<0} - \underbrace{(x^2+1)g}_{<0}$ $\quad \lambda g < 0 \quad \therefore \lambda < 0$

min: $g < 0$, $\frac{d^2g}{dx^2} > 0$

$\lambda g = \underbrace{\frac{d^2g}{dx^2}}_{>0} - \underbrace{(x^2+1)g}_{<0}$ $\quad \lambda g > 0 \quad \therefore \lambda < 0$

no $(\lambda, g)$ exists → stable

(b) $\lambda g = \frac{d^2g}{dx^2} + (x^2+1)g$ $\quad (x^2+1) > 0$ on $[0,1]$

max: $\lambda g = \underbrace{\frac{d^2g}{dx^2}}_{<0} + \underbrace{(x^2+1)g}_{>0}$ $\quad \lambda g$ can be $>, <, = 0$

min: $\lambda g = \underbrace{\frac{d^2g}{dx^2}}_{>0} + \underbrace{(x^2+1)g}_{>0}$ $\quad \lambda g$ can be $>, <, = 0$

can't tell.

(c) $\lambda g = \frac{d^2g}{dx^2} + (x^2-1)g$ $\quad x^2-1=0$ at $x=1$, $x^2-1<0$ on $[0,1)$

max: $\lambda g = \underbrace{\frac{d^2g}{dx^2}}_{>0} + \underbrace{(x^2-1)g}_{<0}$ $\quad \lambda < 0$ $\qquad$ min: $\lambda g = \underbrace{\frac{d^2g}{dx^2}}_{<0} + \underbrace{(x^2-1)g}_{<0}$ $\quad \lambda < 0$

If $x=1$: max: $\lambda g = \underbrace{\frac{d^2g}{dx^2}}_{>0}$ $\quad \lambda < 0$ $\qquad$ min: $\lambda g = \underbrace{\frac{d^2g}{dx^2}}_{<0}$ $\quad \lambda < 0$

no $(\lambda, g)$ exists → stable

(d) $\lambda g = \frac{d^2g}{dx^2} + (2x^2-1)g$

can't tell because $(2x^2-1)$ changes signs on $[0,1]$