Math 19. Lecture 13
Remarks about Australian Predators

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1 A Model

Why there are no large mammalian predators in Australia? Let

$$\begin{aligned} k(t) &= \text{kangaroos at time } t \\ p(t) &= \text{predators at time } t. \end{aligned}$$

Derive the following system,

$$\begin{aligned} \frac{dk}{dt} &= \alpha k - \beta k^2 - \gamma k p \\ \frac{dp}{dt} &= -\sigma p + \lambda k p. \end{aligned}$$

This is a basic predator-prey system with logistic growth for the kangaroos. We have the following null clines.

- The k null clines are

$$\begin{aligned} k &= 0 \\ p &= (\alpha - \beta k)/\gamma. \end{aligned}$$

- The p null clines are

$$\begin{aligned} p &= 0 \\ k &= \sigma/\lambda. \end{aligned}$$

We have two cases.

- $\alpha/\beta < \sigma/\lambda$
- $\alpha/\beta > \sigma/\lambda$

2 Calculating D

$$D = \begin{pmatrix} \frac{\partial}{\partial k}(\alpha k - \beta k^2 - \gamma kp) & \frac{\partial}{\partial p}(\alpha k - \beta k^2 - \gamma kp) \\ \frac{\partial}{\partial k}(-\sigma p + \lambda kp) & \frac{\partial}{\partial p}(-\sigma p + \lambda kp) \end{pmatrix}$$
$$= \begin{pmatrix} \alpha - 2\beta k - \gamma p & -\gamma k \\ \lambda p & -\sigma \quad +\lambda k \end{pmatrix}$$

Case 1. $\alpha/\beta < \sigma/\lambda$ At $(\alpha/\beta, 0)$,

$$D = \begin{pmatrix} -\alpha & -\gamma\alpha/\beta \\ 0 & -\sigma + \lambda\alpha/\beta \end{pmatrix}.$$

Since $\alpha > 0$ we know that

$$\det(D) = \alpha\sigma - \frac{\lambda\alpha^2}{\beta} \text{ if and only if } \frac{\sigma}{\lambda} > \frac{\alpha}{\beta}$$
$$\operatorname{tr}(D) = -\alpha - \sigma + \frac{\lambda\alpha}{\beta} < -\alpha - \sigma + \lambda\left(\frac{\sigma}{\lambda}\right) = -\alpha < 0$$

In this case, $k = \alpha/\beta$ and $p = 0$ is stable, and we are modeling large mammalian predators.

Case 2. $\alpha/\beta > \sigma/\lambda$ At $k = \sigma/\lambda$ and $p = \alpha/\gamma - \beta\sigma/(\lambda\gamma)$

$$D = \begin{pmatrix} -\frac{\beta\sigma}{\lambda} & -\frac{\gamma\sigma}{\lambda} \\ \frac{\lambda\alpha - \beta\sigma}{\gamma} & 0 \end{pmatrix}$$

$$\det(D) = \frac{\sigma}{\lambda}(\lambda\alpha - \beta\sigma)$$
$$\operatorname{tr}(D) = -\frac{\beta\sigma}{\lambda} < 0,$$

Since $\lambda\alpha > \beta\sigma$. From the previous argument, $k = \alpha/\sigma$, $p = 0$ is unstable. In this case, $p > 0$ and $k > 0$ must be the large reptile case.

Readings and References

- C. Taubes. *Modeling Differential Equations in Biology.* Prentice Hall, Upper Saddle River, NJ, 2001. Chapter 12.

- "The Case of the Missing Meat Eaters," pp. 181–184.