Math 19. Mathematical Modeling
Reviewing for Midterm I

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Exam Topics

The exam will cover Chapter 1–11 in *Modeling Differential Equations in Biology*. For this exam you should

- Review and understand the needed concepts from calculus.
- Understand the concept of a differential equation and what it means to be a solution to a differential equation.
- Understand exponential growth and that the growth equation

$$\frac{dx}{dt} = ax$$

  is solved by $x(t) = x(t_0)e^{a(t-t_0)}$, where t_0 is any convenient time.

- Understand the generalization of exponential growth,

$$\frac{dx}{dt} = ax + c$$

  is solved by

$$x(t) = -\frac{c}{a} + \left(x(t_0) + \frac{c}{a}\right) e^{a(t-t_0)},$$

  where t_0 is any convenient time.

- Understand that complicated functions can often be approximated by their first or second order Taylor polynomial near a point.
- Beware of sums of exponential functions. One term or the other will eventually dominate the expression.
- Understand how the logistic equation can be used to model population growth in an environment with limited resources and be able to explain how the qualitative behavior of the solutions $p(t)$ to the logistic equation and the generic equation

  $$\frac{dp}{dt} = f(p)$$

  can be obtained from the graph of f.
- Understand that any equation of the form

  $$\frac{dp}{dt} = f(p)$$

  is completely predictive. If we choose any initial value for p, then there is precisely one solution that starts at this value.
- Understand that an equilibrium point is a value, say x_0, for which the constant function $x(t) = x_0$ solves the equation

  $$\frac{dx}{dt} = f(x).$$

  Equilibrium points correspond to those points for which $f(x) = 0$.
- Understand and be able to determine if an equilibrium point is stable or unstable and be able to explain why equilibrium points are rarely seen in nature.
- Understand and be able to set up systems of first-order differential equations.
- Understand and be able to analyze first-order systems by examining the phase plane.
- Understand and be able to analyze the model for two competing species and the predator-prey model.

- Understand and be able to model an epidemic.
- Understand and to be able to apply phase plane analysis to

$$\begin{aligned} \frac{dx}{dt} &= f(x,y), \\ \frac{dy}{dt} &= g(x,y). \end{aligned}$$

- Understand that the system

$$\begin{aligned} \frac{dx}{dt} &= f(x,y), \\ \frac{dy}{dt} &= g(x,y), \end{aligned}$$

  is completely predictive. If you choose a starting point in the xy-plane, then there is exactly one solution that starts at your chosen point.
- Understand the concept of a vector and be able to apply vectors and vector notation to the representation of systems of differential equations.
- Understand and be able to apply the condition for stability in two-component linear and nonlinear systems.
- Understand and be able to compute the trace and determinant of a 2×2 matrix.
- Understand that the stability question is important since unstable solutions rarely occur in nature.
- Understand partial derivatives and to know how to compute them.
- Understand the concept of a matrix and be able to perform basic operations with matrices.
- Understand eigenvalues and eigenvectors for 2×2 matrices. You will *not* be asked to compute eigenvalues or eigenvectors on this exam
- Be able to apply eigenvalues and eigenvectors to the solution of 2×2 linear system.
- Understand and be able to apply the Principle of Superposition.

Suggested Review Problems

The following should help you prepare for the first exam.

- Go over all old homework assignments and solutions.
- Try some of the extra problems for Chapters 1–12 (pp. 481–488 in the textbook).