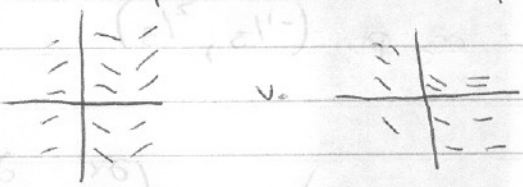
Math 19 Review

- know calculus basics
- what is a differential equation?
  - be able to check possible solution
  - understant that differential equation is a rate

ex. is $x(t) = a + e^{t^2}$ a solution of $\frac{dx}{dt} = tx$

given two slope fields identify which one matches equation



$x(0)e^{at} = x(t)$ if $\frac{dx}{dt} = ax$

- autonomous differential equation: one variable on right
  ex: $\frac{dx}{dt} = f(x)$ -- totally predictive
  - this is when Taylor series are helpful
- understand models: competing species, predator-prey
  - be able to identify (not memorize) & tell what is happening, what constants mean (etc)
  - May be asked to model a given situation

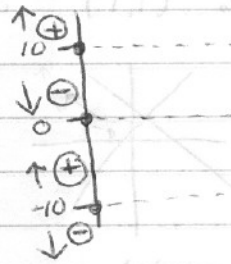
$\frac{dx}{dt} = x(x-10)(x+10)$

phase line:

$\frac{dx}{dt} = 0$ @ $x = 0, 10, -10$

$x = 0$ is stable

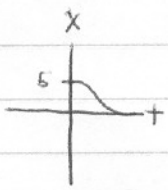
$x = \pm 10$ unstable



$x(0) = 5$

$x(t) \to 0$

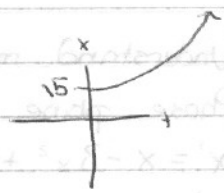
as $t \to \infty$



$x(0) = 15$

$x(t) \to \infty$

as $t \to \infty$



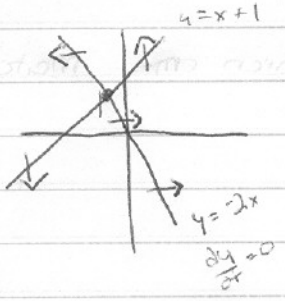
You may use calculators, but will not need much, but may use some

- be able to use phase plane

$x' = x - y + 1$ Null clines: x null cline $y = x + 1$

$y' = 2x + y$ y Null clines ~~$x = -1/2$~~ $y = -2x$

on exam, Null clines will be straight lines



eq. pt. $(-1/3, 2/3)$

stability: $D = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{pmatrix}$ also called Hessian Matrix

$\text{trace}(D) = 1+1 = 2 > 0 \therefore$ not stable

$\det(D) = 1+2 = 3 > 0$

{ stable if $\text{tr}D < 0$ & $\det D > 0$

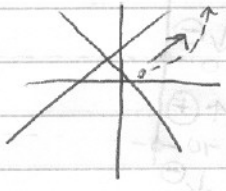
direction of movement on null cline:

only verticle on $\frac{dx}{dt} = 0$ ex: (0,0) in $\frac{dx}{dt} = 1 \therefore \rightarrow$

only horizontal on $\frac{dy}{dt} = 0$

what happens in each quadrant

ex. (1,1) $\frac{dx}{dt} = 1$, $\frac{dy}{dt} = 3$ ↗



initial condition at $(\frac{1}{10}, \frac{1}{10})$ what is trajectory

Understand model epidemics.

Phase plane analysis

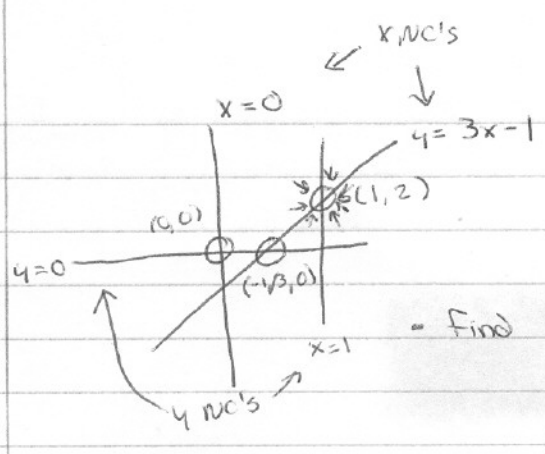
$x' = x - 3x^2 + xy$

$y' = y - xy$

- find null clines x Nc's $= x(1 - 3x + y)$; $x = 0$ & $y = 3x - 1$

  y Nc's $= y(1 - x)$; $y = 0$ & $x = 1$

- graph Null clines



(0,0), (1,2), (-1/3, 0)

- find eq points (should get nice numbers maybe some fractions)
- find stability $D = \begin{pmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{pmatrix}$

$D = \begin{pmatrix} 1-6x+y & x \\ -y & 1-x \end{pmatrix}$ $D(1,2) = \begin{pmatrix} -3 & 1 \\ -2 & 0 \end{pmatrix}$

tr = -3
det = 2 ∴ stable

if (1,2) is stable area near (1,2) will be directed to equilibrium

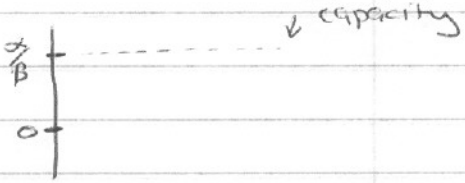
- partial derivatives: know sin, cos, e^x, ln (basic ones)

logistic growth - set up phase line

$dP/dt = \alpha P - \beta P^2$

$P(\alpha - \beta P)$

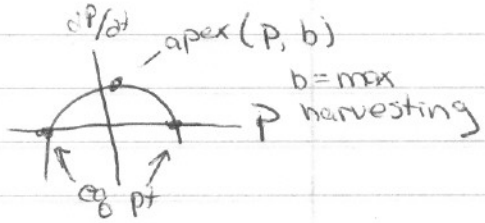
$P = 0, \ \alpha/\beta$ = eq pt. P carring capacity



Harvesting:

$dP/dt = \alpha P - \beta P^2 - C$ ← harvesting

will give parabola when looking for eq pts.



No = multiple integration, eigen values, principle of super position, ∂ field / P plane

Know:
- doubling time & half lives of exponential growth (in book)
- if birth rate & death rate are constant

dP/dt = (birth - death)P

- why in $x' = \alpha x - \beta xy$ β & λ may be different

$y' = -\gamma y + \lambda xy$

β = how often prey is caught

λ = how much benefit predator receives from prey

γ = death rate if no prey

α = growth rate of prey