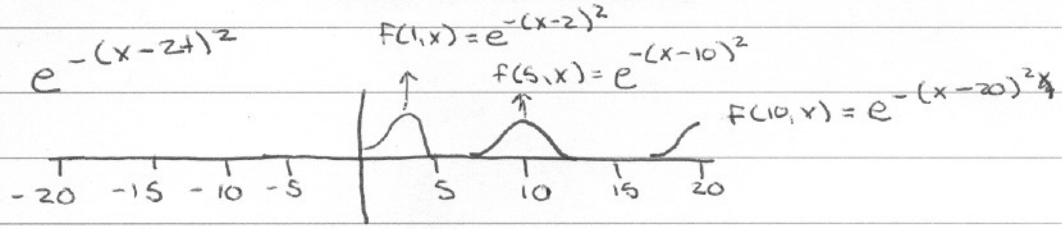
1. $f(t,x) = \sin(x-3t)$ $\quad \dfrac{\partial f}{\partial t} = -3\cos(x-3t)$ $\quad \dfrac{\partial f}{\partial x} = \cos(x-3t)$

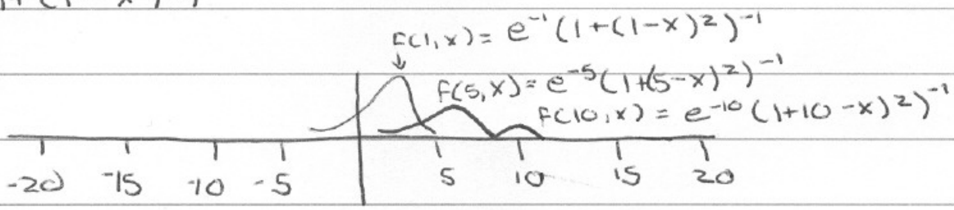
3. $f(t,x) = t^{-1/2} e^{-x^2/t}$ $\quad \dfrac{\partial f}{\partial t} = \left(-\frac{1}{2} t^{-3/2}\right)\left(e^{-x^2/t}\right) + \left(t^{-1/2}\right)\left(e^{-x^2/t} x^2 t^{-2}\right)$

$\dfrac{\partial f}{\partial x} = \left(t^{-1/2} e^{-x^2/t}\right)\left(\dfrac{-2x}{t}\right)$

5. $f(t,x) = e^{-(x-2t)^2}$



6. $f(t,x) = e^{-t}(1+(t-x)^2)^{-1}$



8. (a) $f(t,x) = e^{-(x-2t)^2}$ $\quad c = ?\quad r = ?$

(5) $u(t,x) = e^{-rt} f(x-ct)$

in this case $f(x-ct) = e^{-(x-2t)^2}$ or $f(a) = e^{-a^2}$

so $u(t,x) = e^{-rt} e^{-(x-2t)^2} = e^{-(x-2t)^2}$

$\therefore -rt = 0$ so $r = 0$

and $f(x-ct) = e^{-(x-ct)^2} = e^{-(x-2t)^2}$

so $c = 2$

$$\boxed{r = 0;\ c = 2}$$

also: $\dfrac{\partial u}{\partial t} = -c\dfrac{\partial u}{\partial x} - ru = -c\dfrac{\partial u}{\partial x}$

$\partial u/\partial t = e^{-(x-2t)^2}(-2(x-2t)(-2)) = 4(x-2t)e^{-(x-2t)^2}$

$\partial u/\partial x = e^{-(x-2t)^2}(-2)(x-2t) = -2(x-2t)e^{-(x-2t)^2}$ continued:

$$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x} \qquad c=2$$

$$4(x-2t)e^{-(x-2t)^2} = (-2)\left(-2(x-2t)e^{-(x-2t)^2}\right) \checkmark$$

(6) $u = e^{-t}(1+(t-x)^2)^{-1} = e^{-rt} f(x-ct)$

so $e^{-t} = e^{-rt}$

$r=1$

$(1+(t-x)^2)^{-1} = f(x-ct)$

if $f(a) = (1+(-a)^2)^{-1}$

then $f(x-ct) = (1+(-x+ct)^2)^{-1} = (1+(ct-x)^2)^{-1} \Rightarrow (1+(t-x)^2)^{-1}$

$\boxed{\text{so } c=1;\ r=1}$

check: $\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x} - ru$

$u_t = -u_x - u$

$u_t = -e^{-t}(1+(t-x)^2)^{-1} - (1+(t-x)^2)^{-2}(2(t-x))(e^{-t})$

$u_x = -e^{-t}(1+(t-x)^2)^{-1}(-2)(t-x)$ $\checkmark$

so: $-e^{-t}(1+(t-x)^2)^{-1} - 2e^{-t}(1+(t-x)^2)^{-2}(t-x) = e^{-t}(1+(t-x)^2)^{-1}(-2)(t-x) - e^{-t}(1+(t-x)^2)$

(7) $f(t,x) = t^{-1/2}e^{-x^2/t}$ find μ if $u_t = \mu u_{xx}$

$u_t = \left(-\tfrac{1}{2}t^{-3/2}\right)\left(e^{-x^2/t}\right) + \left(t^{-1/2}\right)\left(e^{-x^2/t}\right)\left(+\frac{x^2}{t^2}\right)$

$u_x = \left(t^{-1/2}\right)\left(e^{-x^2/t}\right)\left(-\frac{2x}{t}\right)$

$u_{xx} = \left(t^{-1/2}\right)\left(-\frac{2}{t}\right)\left(e^{-x^2/t}\right) + \left(t^{-1/2}\right)\left(\frac{-2x}{t}\right)\left(e^{-x^2/t}\right)\left(\frac{-2x}{t}\right)$

$= -2t^{-3/2}\left(e^{-x^2/t}\right) + 4x^2t^{-5/2}e^{-x^2/t}$

$u_t = \mu u_{xx}$:

$-\frac{1}{2}t^{-3/2}\left(e^{-x^2/t}\right) + x^2t^{-5/2}e^{-x^2/t} = \left(-2t^{-3/2}e^{-x^2/t} + 4x^2t^{-5/2}e^{-x^2/t}\right)\mu$

$\left(-\tfrac{1}{2}A + 1B\right) = \mu(-2A + 4B)$

$-\tfrac{1}{2}A = \mu\, {-2A}$; $B = \mu 4B$

$\mu = 1/4$; $\mu = 1/4$

$\boxed{\mu = \frac{1}{4}}$

10 (a) $e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} = u$

$u_t = \lambda e^{\lambda t} e^{x(\lambda/\mu)^{1/2}}$

$u_x = e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} (\lambda/\mu)^{1/2}$

$u_{xx} = e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} ((\lambda/\mu)^{1/2})^2 = e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} (\lambda/\mu)$

$u_t = \mu u_{xx}$

$\lambda e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} = \cancel{\mu}\left(\frac{\lambda}{\cancel{\mu}}\right) e^{x(\lambda/\mu)^{1/2}} e^{\lambda t}$ ✓

(c) $u = a + bx$

$u_t = 0$

$u_x = b$

$u_{xx} = 0$

$u_t = u_{xx}(\mu)$

$0 = 0$ ✓

(e) $e^{-\lambda t} \sin((\lambda/\mu)^{1/2} x) = u$

$u_t = -\lambda e^{-\lambda t} \sin((\lambda/\mu)^{1/2} x)$

$u_x = e^{-\lambda t} (\lambda/\mu)^{1/2} \cos((\lambda/\mu)^{1/2} x)$

$u_{xx} = -e^{-\lambda t} (\lambda/\mu) \sin((\lambda/\mu)^{1/2} x)$

$u_t = u_{xx} \mu$

$-\lambda e^{-\lambda t} \sin\left(\left(\frac{\lambda}{\mu}\right)^{1/2} x\right) = \mu\left(\frac{\lambda}{\mu}\right)\left(-e^{-\lambda t} \sin\left(\left(\frac{\lambda}{\mu}\right)^{1/2} x\right)\right)$ ✓

12. (a) $u = e^{\lambda t} e^{x((\lambda - r)/\mu)^{1/2}}$

$u_t = \lambda e^{\lambda t} e^{x((\lambda - r)/\mu)^{1/2}}$

$u_x = e^{\lambda t} \left(\frac{\lambda - r}{\mu}\right)^{1/2} e^{x((\lambda - r/\mu))^{1/2}}$

$u_{xx} = e^{\lambda t} \left(\frac{\lambda - r}{\mu}\right) e^{x((\lambda - r)/\mu)^{1/2}}$

$u_t = u_{xx} \mu + ru$

$\lambda e^{\lambda t} e^{x((\lambda - r)/\mu)^{1/2}} =$

$e^{\lambda t} \mu \left(\frac{\lambda - r}{\mu}\right)\left(e^{x((\lambda - r)/\mu)^{1/2}}\right) + r e^{\lambda t} e^{x((\lambda - r)/\mu)^{1/2}}$ ✓

(c) $e^{rt}(a + bx)$

$u_t = r e^{rt}(a + bx)$

$u_x = e^{rt} b$

$u_{xx} = 0$

$u_t = u_{xx} \mu + ru$

$r e^{rt}(a + bx) = 0 \; r e^{rt}(a + bx)$

12 (e) $u = e^{\lambda t} \sin\left[((r-\lambda)/\mu)^{1/2} x\right]$

$u_t = \lambda u$

$u_x = e^{\lambda t} \left(\frac{r-\lambda}{\mu}\right)^{1/2} \cos\left[((r-\lambda)/\mu)^{1/2} x\right]$

$u_{xx} = -e^{\lambda t} \left(\frac{r-\lambda}{\mu}\right) \sin\left[\left(\frac{r-\lambda}{\mu}\right)^{1/2} x\right]$

$u_t = \mu u_{xx} + ru$

$\lambda u = (\lambda - r)u + ru$ ✓