

$$(7) \quad \frac{\partial}{\partial t} u(t, x) = -3 \frac{\partial}{\partial x} u(t, x) - r u(t, x)$$

$$(8) \quad u(t, x) = e^{-rt} f(x-3t)$$

$$r = 2$$

$$1. \quad u(t, x) = e^{-2t} \sin(x-3t) \quad \frac{\partial u}{\partial t} = -2e^{-2t} \sin(x-3t) - 3e^{-2t} \cos(x-3t)$$

$$\frac{\partial u}{\partial x} = e^{-2t} \cos(x-3t)$$

$$(7) \quad -2e^{-2t} \sin(x-3t) - 3e^{-2t} \cos(x-3t) = -3(e^{-2t} \cos(x-3t)) - 2(e^{-2t} \sin(x-3t))$$

$$\frac{\partial u}{\partial t} = -3 \frac{\partial u}{\partial x} - r u$$

$$3. \quad u(t, x) = e^{-2t} e^{-2(x-3t)/3} = e^{-2t - 2/3x + 2t} = e^{-2/3x}$$

$$\frac{\partial u}{\partial t} = 0 \quad \frac{\partial u}{\partial x} = -2/3 e^{-2/3x}$$

$$\frac{\partial u}{\partial t} = -3 \frac{\partial u}{\partial x} - r u$$

$$0 = -3(-2/3 e^{-2/3x}) - 2 e^{-2/3x} \\ = 2e^{-2/3x} - 2e^{-2/3x} = 0$$

or. if  $u(t, x) = e^{-2x}$  (the book is wrong)

$$\frac{\partial u}{\partial t} = 0 \quad \frac{\partial u}{\partial x} = -2e^{-2x}$$

$$0 \neq -3(-2e^{-2x}) - 2e^{-2x}$$

$0 \neq 6e^{-2x} - 2e^{-2x}$  so  $e^{-2x} = u(t, x)$  is not a solution

$$5. \quad u(t, x) = e^{-2t} (1 + (x-3t)^2)^{-1}$$

$$\frac{\partial u}{\partial t} = -2e^{-2t} (1 + (x-3t)^2)^{-1} + e^{-2t} (-3 \cdot 2(x-3t)) (-1)(1 + (x-3t)^2)^{-2} \\ = -2e^{-2t} (1 + (x-3t)^2)^{-1} + e^{-2t} (6(x-3t)(1 + (x-3t)^2)^{-2}$$

$$\frac{\partial u}{\partial x} = -1(1 + (x-3t)^2)^{-2} (2(x-3t)) e^{-2t} \quad \left| \quad \frac{\partial u}{\partial t} = -3 \frac{\partial u}{\partial x} - r u \quad \checkmark \quad \text{by inspection} \right.$$

$$r = 2$$

$$6. u(0, x) = \cos(x)$$

$$u(0, x) = e^{-rt} f(x-3t) = \cos(x) = f(x)$$

$$\text{so, } u(t, x) = e^{-2t} \cos(x-3t)$$

$$7. u(0, x) = e^{-4x}$$

$$u(0, x) = e^{-r(0)} f(x-0) = f(x) = e^{-4x} \Rightarrow f(x-3t) = e^{-4(x-3t)}$$

$$u(t, x) = e^{-2t} e^{-4x+12t} = e^{-4x+10t} = e^{10t} e^{-4x} = e^{10t} / e^{4x}$$

$$8. u(0, x) = (1+x^2)^{-1}$$

$$u(0, x) = e^{-r(0)} f(x-0) = f(x) = (1+x^2)^{-1} \Rightarrow f(x-3t) = (1+(x-3t)^2)^{-1}$$

$$u(t, x) = e^{-2t} (1+(x-3t)^2)^{-1}$$