Problem Set # 6 Key

pg. 138 - 139 #'s 1, 2, 3, 4, 5, 7

pg. 145 - 146 #'s 1, 2, 4, 5

1. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x + 4y \\ y \end{pmatrix}$

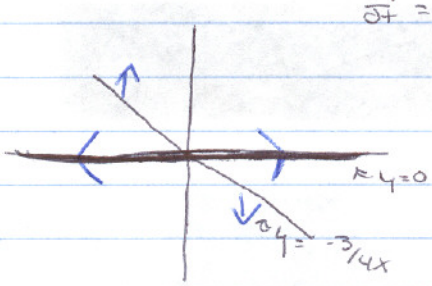
x null cline

$\frac{dx}{dt} = 0 = 3x + 4y$

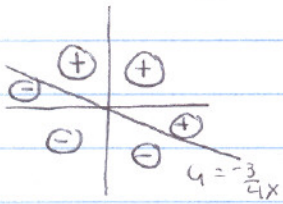
$y = -\frac{3}{4}x$

y null cline

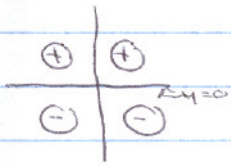
$\frac{dy}{dt} = 0 = y$



2. for $\frac{dx}{dt} = 3x + 4y$



for $\frac{dy}{dt} = y$



3. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - 2y \\ x + y \end{pmatrix}$

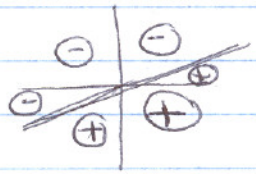
x nullcline $\frac{dx}{dt} = 0$ $y = \frac{1}{2}x$

y nullcline $\frac{dy}{dt} = 0$ $y = -x$

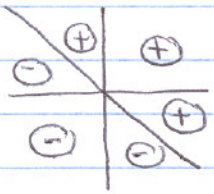
intersection of nullclines = intersection of null clines

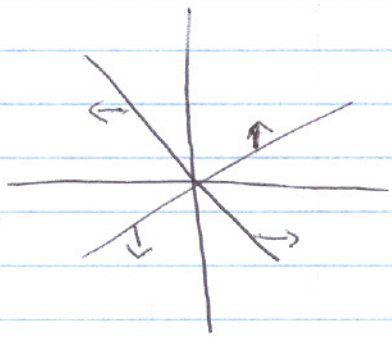
(0, 0)

4. $\frac{dx}{dt} = x - 2y$



$\frac{dy}{dt} = x + y$

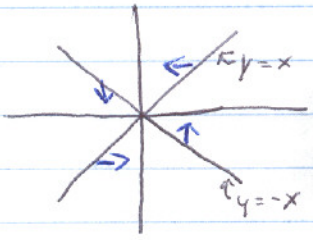




5. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x & -y \\ x & -y \end{pmatrix}$

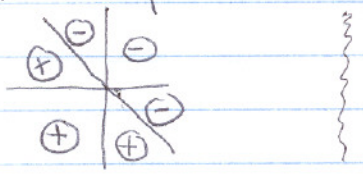
x Null cline $\frac{dx}{dt} = 0$ $y = -x$

y Null cline $\frac{dy}{dt} = 0$ $y = x$

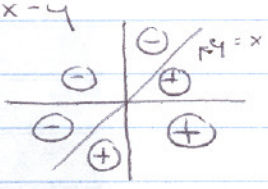


eq. pt @ (0,0)

$\frac{dx}{dt} = -x-y$



$\frac{dy}{dt} = x-y$



7. for eq. 1

$D = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial f}{\partial y} \end{pmatrix}$ $\frac{dx}{dt} = \begin{pmatrix} f(x) \\ g(x) \end{pmatrix}$

$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x+4y \\ y \end{pmatrix}$ $D = \begin{pmatrix} 3 & 4 \\ 0 & 1 \end{pmatrix}$

$tr(A) = 4 > 0$ ∴ unstable

$det(D) = 3$

q. 3 $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-2y \\ x+y \end{pmatrix}$ $D = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$

$tr(D) = 2 > 0$ ∴ unstable

$det(D) = 1+2 = 3$

q. 5 $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x & -y \\ x & -y \end{pmatrix}$ $D = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$

$tr(D) = -2 < 0$

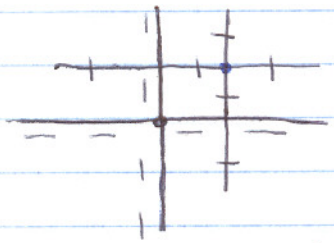
$det(D) = 1+1 > 0$ ∴ stable

p. 145

1. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xy - 2x \\ xy - 2y \end{pmatrix}$

x Null clines
$\frac{dx}{dt} = 0 = x(y-2)$
$x = 0$; $y = 2$

y Null cline
$\frac{dy}{dt} = 0 = y(x-2)$
$y = 0$; $x = 2$



eq pts: (0,0) stable
~~(2,0) unstable~~
(2,2) unstable

$D = \begin{pmatrix} y-2 & x \\ y & x-2 \end{pmatrix}$

at (0,0) $D = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$
$tr(D) = -4 < 0$
$\det(D) = 4 > 0$ ∴ stable

~~at (2,0) $D = \begin{pmatrix} -2 & 2 \\ 0 & 0 \end{pmatrix}$~~
~~$tr(D) = -2 < 0$~~
~~$\det(D) = 0$ unstable~~

at (2,2) $D = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$
$tr(D) = 0$; $\det(D) = -4$ unstable

2. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - x^2 - xy \\ y - 2xy - 2y^2 \end{pmatrix}$

x nullcline = $x(1-x-y) = 0$; $x = 0$, $y = 1-x$
y nullcline = $y(1-2x-2y) = 0$; $y = 0$, $y = \frac{1}{2} - x$

eq points: (0,0) un. $y = 1-x$ } parallel
(1,0) stable $y = \frac{1}{2} - x$ } No intersec
$(0, \frac{1}{2})$ un.

$D = \begin{pmatrix} 1-2x-y & -x \\ -2y & 1-2x-4y \end{pmatrix}$

(0,0) $D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$tr\,D = 2 > 0$; $\det D = 1 > 0$ unstable

(1,0) $D = \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix}$
$tr\,D = -2 < 0$
$\det D = 1 > 0$ Stable

$(0, \frac{1}{2})$ $D = \begin{pmatrix} \frac{1}{2} & 0 \\ -1 & -1 \end{pmatrix}$
$tr\,D = -\frac{1}{2} < 0$
$\det D = -\frac{1}{2} < 0$ ustable

4.

(a) $h(x,y) = xy^3 - x^2y^2$

$\frac{\partial h}{\partial x} = y^3 - 2xy^2$ $\quad$ $\frac{\partial h}{\partial y} = 3xy^2 - 2x^2y$

(b) $h(x,y) = \cos(xy)$

$\frac{\partial h}{\partial x} = -y\sin(xy)$ $\quad$ $\frac{\partial h}{\partial y} = -x\sin(xy)$

(c) $h(x,y) = x\cos(y) - 1$

$\frac{\partial h}{\partial x} = \cos(y)$ $\quad$ $\frac{\partial h}{\partial y} = -x\sin(y)$

(d) $h(x,y) = y\cos(x) - x^2$

$\frac{\partial h}{\partial x} = -y\sin(x) - 2x$ $\quad$ $\frac{\partial h}{\partial y} = \cos(x)$

5. $x(t) = \cos(t)$ $\quad$ $y(t) = \sin(t)$

a.a. $h(x,y) = \cos(t)\sin^3(t) - \cos^2(t)\sin^2(t)$

a.b. $h(x,y) = \cos(\cos(t)\sin(t))$

a.c. $h(x,y) = \cos(t)\cos(\sin(t)) - 1$

a.d. $h(x,y) = \sin(t)\cos(\cos(t)) - \cos^2(t)$

b.a. $h(x,y) = \cos(t)\sin^3(t) - \cos^2(t)\sin^2(t)$

$\frac{dh}{dt} = -\sin^4(t) + 3\sin^2(t)\cos^2(t) + 2\cos(t)\sin^3(t) - 2\sin(t)\cos^3(t)$

b.b. $h(x,y) = \cos(\cos(t)\sin(t))$

$\frac{dh}{dt} = (-\sin^2(t) + \cos^2(t))(-\sin(\cos(t)\sin(t)))$

b.c. $h(x,y) = \cos(t)\cos(\sin(t)) - 1$

$\frac{dh}{dt} = -\sin(t)\cos(\sin(t)) + -\cos^2(t)\sin(\sin(t))$

b.d. $h(x,y) = \sin(t)\cos(\cos(t)) - \cos^2(t)$

$\frac{dh}{dt} = \cos(t)\cos(\cos(t)) + + \sin^2(t)\sin(\cos(t)) + 2\cos(t)\sin(t)$

c. chain rule: $\frac{\partial x}{\partial t} \cdot \frac{\partial h}{\partial x} + \frac{\partial y}{\partial t} \cdot \frac{\partial h}{\partial y} = \frac{\partial h}{\partial t}$

$\frac{\partial x}{\partial t} = -\sin(t)$ $\quad$ $\frac{\partial y}{\partial t} = \cos(t)$

c.a. $\frac{\partial h}{\partial x} = y^3 - 2xy^2$ $\quad$ $\frac{\partial h}{\partial y} = 3xy^2 - 2x^2y$

$\frac{\partial h}{\partial t} = -\sin(t)(\sin^3(t) - 2\cos(t)\sin^2(t) + \cos(t)(3\cos(t)\sin^2(t) - 2\cos^3(t)\sin(t))$

$\frac{\partial x}{\partial t} = -\sin(t)$ $\qquad$ $\frac{\partial y}{\partial t} = \cos(t)$

c.b. $\frac{\partial h}{\partial x} = -y\sin(xy)$ $\qquad$ $\frac{\partial h}{\partial y} = -x\sin(xy)$

$\frac{dh}{dt} = -\sin(t)\left(-\cancel{\cos}\sin(t)\sin(\cos(t)\sin(t))\right) - \cos(t)\left(\cos(t)\sin(\cos(t)\sin(t))\right)$

c.c. $\frac{\partial h}{\partial x} = \cos(y)$ $\qquad$ $\frac{\partial h}{\partial y} = -x\sin y$

$\frac{dh}{dt} = -\sin(t)\cos(\sin(t)) - \cos^2(t)\sin(\sin t)$

c.d. $\frac{\partial h}{\partial t} = -y\sin(x) - 2x$ $\qquad$ $\frac{\partial h}{\partial y} = \cos(x)$

$\frac{dh}{dt} = -\sin(t)\left(-\sin(t)\sin(\cos(t)) - 2\cos(t)\right) + \cos(t)\cos(\cos(t))$