Problem Set #5 Key

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Part 1 #1

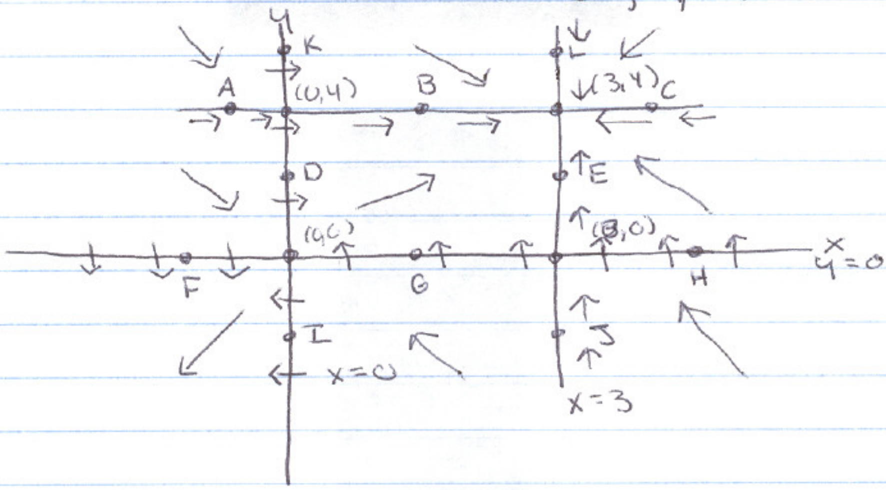
Part 2 # 1-6

1-1. $\frac{dx}{dt} = 3y - xy$ x Null cline $\frac{dx}{dt} = 0 = y(3-x)$

$\frac{dy}{dt} = 4x - xy$ $y = 0$; $x = 3$

y Null cline $\frac{dy}{dt} = 0 = x(4-y)$

$x = 0$; $y = 4$



Find Direction:

A: $(-1,4)$ in $\frac{dx}{dt} = 3(4) - (-1)(4) = 12 + 4 = 16$ ⊕ →

B: $(2,4)$ in $\frac{dx}{dt} = 3(4) - (2)(4) = 12 - 8 = 4$ ⊕ →

C: $(4,4)$ in $\frac{dx}{dt} = 3(4) - (4)(4) = 12 - 16 = -4$ ⊖ ←

D: $(0,2)$ in $\frac{dx}{dt} = 3(2) - 0 = 6$ ⊕ →

E: $(3,2)$ in $\frac{dy}{dt} = 4(3) - (3)(2) = 12 - 6 = 6$ ⊕ ↑

F: $(-1,0)$ in $\frac{dy}{dt} = 4(-1) - 0 = -4$ ⊖ ↓

G: $(2,0)$ in $\frac{dy}{dt} = 4(2) - 0 = 8$ ⊕ ↑

H: $(4,0)$ in $\frac{dy}{dt} = 4(4) - 0 = 16$ ⊕ ↑

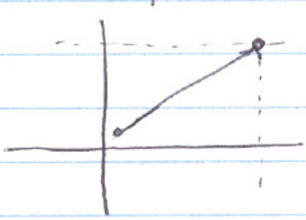
I: $(0,-1)$ in $\frac{dx}{dt} = 3(-1) - 0 = -3$ ⊖ ←

J: $(3,-1)$ in $\frac{dy}{dt} = 4(3) - (-1)(3) = 12 + 3 = 15$ ⊕ ↑

K: $(0,5)$ in $\frac{dx}{dt} = 3(5) - 0 = 15$ ⊕ →

L: $(3,5)$ in $\frac{dy}{dt} = 4(3) - (3)(5) = 12 - 15 = -3$ ⊖ ↓

$x(0), y(0) = 1/10, 1/10$



a trajectory that begins at $(1/10, 1/10)$ will increase to equilibrium at $(3,4)$

2-1.

	v	w	$v+w$	$v-w$
(a)	$\begin{pmatrix}5\\5\end{pmatrix}$	$\begin{pmatrix}-3\\2\end{pmatrix}$	$\begin{pmatrix}2\\7\end{pmatrix}$	$\begin{pmatrix}8\\3\end{pmatrix}$
(b)	$\begin{pmatrix}3\\1\end{pmatrix}$	$\begin{pmatrix}-10\\-31\end{pmatrix}$	$\begin{pmatrix}-7\\-30\end{pmatrix}$	$\begin{pmatrix}13\\32\end{pmatrix}$
(c)	$\begin{pmatrix}2\\0\end{pmatrix}$	$\begin{pmatrix}6\\-1\end{pmatrix}$	$\begin{pmatrix}8\\-1\end{pmatrix}$	$\begin{pmatrix}-4\\1\end{pmatrix}$
(d)	$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}-1\\0\end{pmatrix}$	$\begin{pmatrix}-1\\1\end{pmatrix}$	$\begin{pmatrix}1\\1\end{pmatrix}$

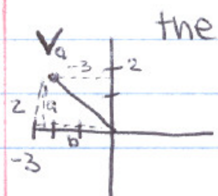
2-2.

	r	v	rv
(a)	3	$\begin{pmatrix}-3\\2\end{pmatrix}$	$\begin{pmatrix}-9\\6\end{pmatrix}$
(b)	1	$\begin{pmatrix}2\\0\end{pmatrix}$	$\begin{pmatrix}2\\0\end{pmatrix}$
(c)	0	$\begin{pmatrix}-10\\-31\end{pmatrix}$	$\begin{pmatrix}0\\0\end{pmatrix}$
(d)	-2	$\begin{pmatrix}3\\1\end{pmatrix}$	$\begin{pmatrix}-6\\-2\end{pmatrix}$

2-3.

	v	$\lvert v\rvert$
(a)	$\begin{pmatrix}-3\\2\end{pmatrix}$	$\sqrt{(3)^2+(2)^2} = \sqrt{9+4} = \sqrt{13} = 3.6$
(b)	$\begin{pmatrix}2\\0\end{pmatrix}$	$\sqrt{(2)^2+0} = 2$
(c)	$\begin{pmatrix}-10\\-31\end{pmatrix}$	$\sqrt{(10)^2+(31)^2} = \sqrt{100+961} = \sqrt{1061} = 32.6$
(d)	$\begin{pmatrix}3\\1\end{pmatrix}$	$\sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10} = 3.2$

because length can be calculated by the pythagorean theorum



$c = \sqrt{a^2+b^2}$

2-4.

a. $\begin{pmatrix}5\\5\end{pmatrix}\cdot\begin{pmatrix}-3\\2\end{pmatrix} = -15+10 = -5$

b. $\begin{pmatrix}3\\1\end{pmatrix}\cdot\begin{pmatrix}-10\\-31\end{pmatrix} = -30-31 = -61$

c. $\begin{pmatrix}2\\0\end{pmatrix}\cdot\begin{pmatrix}6\\-1\end{pmatrix} = 12+0 = 12$

d. $\begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\0\end{pmatrix} = 0$

2-5.

(a) $\mathbf{v}(t) = \begin{pmatrix} \cos(2t) \\ 3e^{-2t} \end{pmatrix}$ $\quad \mathbf{v}'(t) = \begin{pmatrix} -2\sin(2t) \\ -6e^{-2t} \end{pmatrix}$

(b) $\mathbf{v}(t) = \begin{pmatrix} e^{2t} \\ t^2 \end{pmatrix}$ $\quad \mathbf{v}'(t) = \begin{pmatrix} 2e^{2t} \\ 2t \end{pmatrix}$

(c) $\mathbf{v}(t) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\quad \mathbf{v}'(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(d) $\mathbf{v}(t) = \begin{pmatrix} -\sin(t) \\ 2t \end{pmatrix}$ $\quad \mathbf{v}'(t) = \begin{pmatrix} -\cos(t) \\ 2 \end{pmatrix}$

2-6. Compute antiderivative = integrate

(a) $\mathbf{v}(t) = \begin{pmatrix} \cos(2t) \\ 3e^{-2t} \end{pmatrix}$ $\quad \int \mathbf{v}(t)\,dt = \begin{pmatrix} \frac{1}{2}\sin(2t) + C_1 \\ -3/2\, e^{-2t} + C_2 \end{pmatrix}$

$\mathbf{v}(t) = \begin{pmatrix} e^{2t} \\ t^2 \end{pmatrix}$ $\quad \int \mathbf{v}(t)\,dt = \begin{pmatrix} \frac{1}{2}e^{2t} + C_1 \\ \frac{1}{3}t^3 + C_2 \end{pmatrix}$

$\mathbf{v}(t) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\quad \int \mathbf{v}(t)\,dt = \begin{pmatrix} 3t + C_1 \\ t + C_2 \end{pmatrix}$

$\mathbf{v}(t) = \begin{pmatrix} -\sin(t) \\ 2t \end{pmatrix}$ $\quad \int \mathbf{v}(t)\,dt = \begin{pmatrix} \cos(t) + C_1 \\ t^2 + C_2 \end{pmatrix}$