

Problem set # 4 ch 5  
Pg. 86  
ex. 1

1. There are many good answers to this question.  
Here is one.

A. First by observing if one population quickly became extinct, when given an advantage, it would be possible to tell if  $a > 1$ . If the extinction occurs easily  $a > 1$ ; if not,  $a < 1$ .

B. To determine  $a$  more specifically:

Place the same number of R snails + L snails in a test environment. Observe where the population reaches equilibrium.

Count the numbers of R + L snails. Plug these numbers into  $\frac{dR}{dt} + \frac{dL}{dt}$  along with the fact that at equilibrium  $\frac{dR}{dt} + \frac{dL}{dt} = 0$  to solve for  $a$ .

OR

B. Both R + L do not have to reach equilibrium.

For example, if the population of R remains constant you have found an R null cline.  $\therefore$  at this point  $\frac{dR}{dt} = 0$ . Count the number of R + L + plug into  $\frac{dR}{dt}$  to find value of  $a$ .

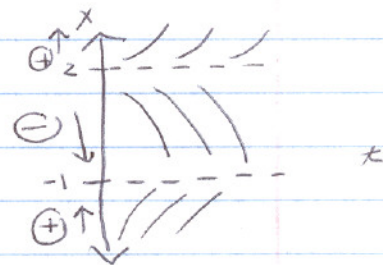
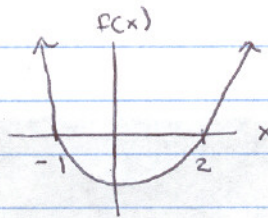
Other solutions may also be correct.

ex. 1, 2, 3, 4 (a,c), 5

1.  $\frac{dx}{dt} = f(x)$ ; describe  $x(t)$  as  $t \rightarrow \infty$  if  $x(0) = 1$

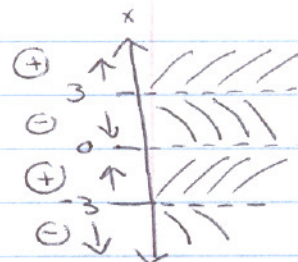
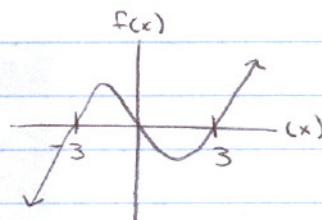
(a).  $f(x) = (x+1)(x-2)$

$x(t)$  will decrease to equilibrium at  $x = -1$  as  $t \rightarrow \infty$



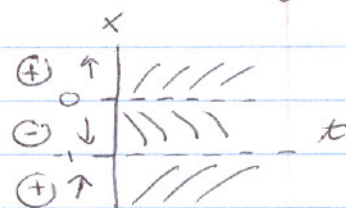
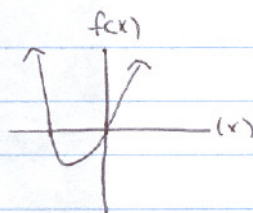
(b).  $f(x) = (x+3)x(x-3)$

$x(t)$  will decrease to equilibrium at  $x = 0$



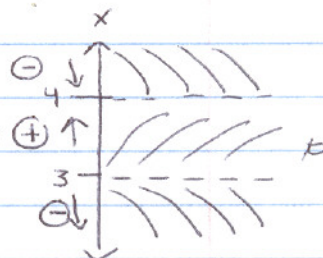
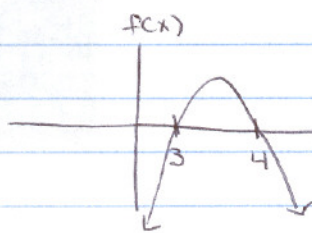
(c).  $f(x) = (x+1)x$

$x(t)$  will increase to  $\infty$



(d).  $f(x) = (3-x)(x-4)$

$x(t)$  will decrease to  $-\infty$



2. describe  $x(t)$  at  $t \rightarrow \infty$  if  $x(0) = -4$

(see # 1 for graphs of  $f(x)$  & phase lines)

(a)  $x(t)$  will increase to equilibrium at  $x = -1$

(b)  $x(t)$  will decrease to  $-\infty$

(c)  $x(t)$  will increase to equilibrium at  $x = -1$

(d)  $x(t)$  will decrease to  $-\infty$

3. see phase lines for # 1

a. 2, unstable; -1 = stable

b. -3 unstable, 0 stable, 3 unstable

c. -1 stable, 0 unstable

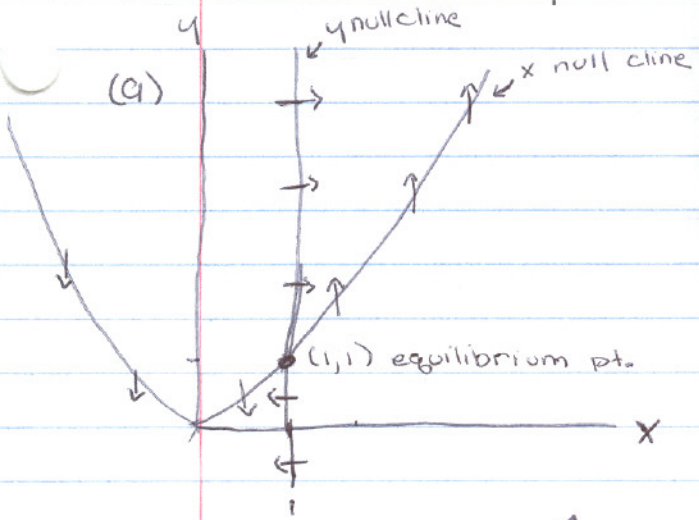
d. 3 unstable, 4 stable

4. Taylor expansion for  $f(x) \approx g(x) = f(a) + f'(a)x$   
 where  $a$  is the value about which you are trying to describe the function  $x_0 = 0$

(a)  $f(x) = x e^{-6x}$      $f'(x) = -6x e^{-6x} + e^{-6x}$   
 $f(0) = 0$      $f'(0) = 0 + 1 = 1$   
 $g(x) = 0 + 1x = x$

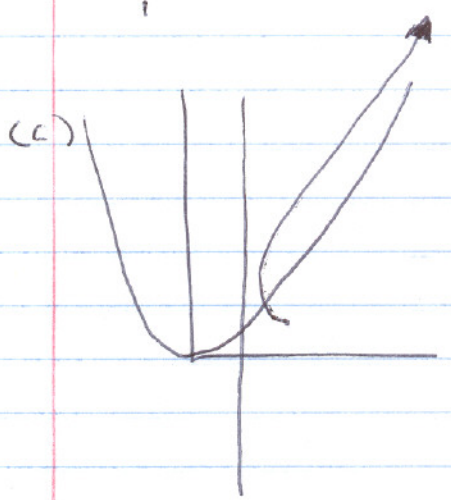
(c)  $f(x) = 4x + x^2$      $f'(x) = 4 + 2x$   
 $f(0) = 0$      $f'(0) = 4 + 0$   
 $g(x) = 0 + 4x = 4x$

5.  $\frac{dx}{dt} = y - x^2$      $x$  null cline  $\frac{dx}{dt} = 0 = y - x^2$      $y = x^2$   
 $\frac{dy}{dt} = x - 1$      $y$  null cline  $\frac{dy}{dt} = 0 = x - 1$      $x = 1$



(b)  $x$  null cline  
 $(2, 4)$  in  $\frac{dy}{dt} = 2 - 1 = 1$  positive = up  
 $(0, 0)$  in  $\frac{dy}{dt} = 0 - 1 = -1$  negative = down

$y$  null cline  
 $(1, 0)$  in  $\frac{dx}{dt} = -1$  negative, left  
 $(1, 2)$  in  $\frac{dx}{dt} = 1$  positive, right



(d)  $x(t) < x(0)$   
 $y(t) < y(0)$