Problem Set #19
Math 19

Chapter 25

1. $\frac{dx}{dt} = 3x + x^2 + 3$ $\quad x(0)=0,\ x(t_1)=1$

$\left.\frac{dx}{dt}\right|_{t=0} = 3$ $\quad \left.\frac{dx}{dt}\right|_{t=1} = 3+1+3 = 7$ $\quad$ let $\frac{dx}{dt} = f(x)$

$f_{min} \le \frac{dx}{dt} \le f_{max}$

$\int_0^{t_1} f_{min}\,dt \le \int \frac{dx}{dt} \le \int_0^{t_1} f_{max}\,dt$

$f_{min} \cdot t_1 \le x(t_1) - x(0) \le f_{max} \cdot t_1$

are there any max/mins on this interval?

$\frac{d}{dx}\left(\frac{dx}{dt}\right) = 3 + 2x = 0$ for max/min

$x = -3/2$ Not in interval $0 \le x \le 1$

$\therefore$ end points = max/min

so: $3 \cdot t_1 \le 1 - 0 \le 7 \cdot t_1$

$\boxed{\frac{1}{7} \le t_1 \le \frac{1}{3}}$

2. $\frac{dx}{dt} = x^3 + 5 = f(x)$ $\quad \frac{dx}{dt}(0) = 5$ $\quad \frac{dx}{dt}(1) = 6$

$f'(x) = 3x^2 = 0$

$x = 0$ already an end point

$f_{min} \cdot t_1 \le 1 - 0 \le f_{max} \cdot t_1$

$5 \cdot t_1 \le 1 \le 6 \cdot t_1$

$\boxed{\frac{1}{6} \le t_1 \le \frac{1}{5}}$

5. $\frac{dx}{dt} = 2x - x^2 + 1 = f(x)$ $\quad \frac{dx}{dt}(0) = 1$ $\quad \frac{dx}{dt}(1) = 2$

$f'(x) = 2 - 2x = 0$

$x = 1$

$1 \cdot t_1 \le 1 \le 2 \cdot t_1$

$\boxed{\frac{1}{2} \le t_1 \le 1}$

6. $\frac{dx}{dt} = 2x + x^2 + 3 = f(x)$ $\qquad$ $\frac{dx}{dt}(0) = 3$ $\quad$ $\frac{dx}{dt}(1) = 6$

$f'(x) = 2 + 2x = 0$ $\qquad$ $f_{min} \cdot t_1 \le 1 \le f_{max} \cdot t_1$

$x = -1$, not in interval $\qquad$ $\boxed{\frac{1}{6} \le t_1 \le \frac{1}{3}}$

Chapter 26

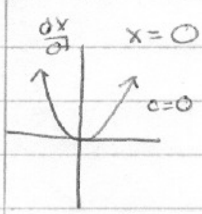
1. (a) $\frac{dx}{dt} = .1x^3 - 3xy \rightarrow$ faster
$\frac{dy}{dt} = .1y - .0001xy \rightarrow$ slower

1. (c) $\frac{dx}{dt} = .1x - .03xy \rightarrow$ slower
$\frac{dy}{dt} = y + .007xy \rightarrow$ faster
$\Big\}$ but this is a close call

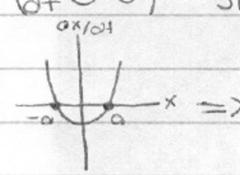
2 (a). $\frac{dx}{dt} = 5x^2 + c = f(x)$

$f'(x) = 10x = 0$

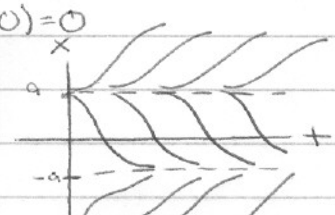
$x = 0$ $\quad$ so $c = -\left(\frac{dx}{dt} @ 0\right) = 5(0) = 0$



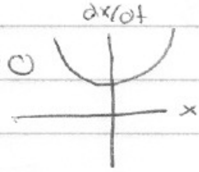
II. if $c < 0$



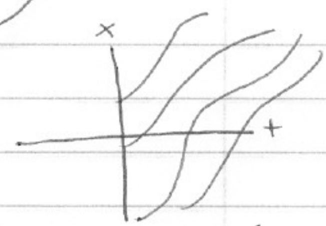
$\Rightarrow$



III. if $c > 0$

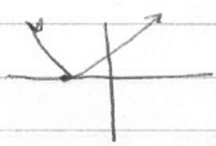


No roots $\therefore$ No eq pts
always increasing

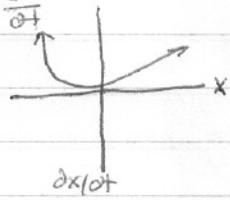


(c) $\frac{dx}{dt} = e^{-x} + cx$

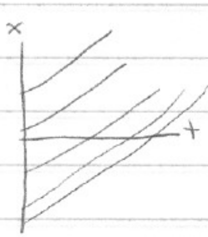
when $c = e$



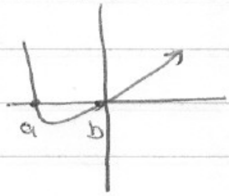
$c < e$



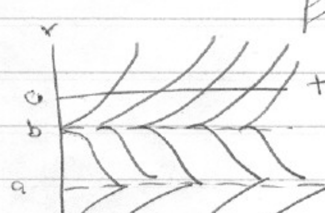
always increasing



$c > e$



$\Rightarrow$



3. temperature sensitive genes, action potentials, heart beat
almost anything with a threshold value.