Problem set #18

math 19

1. (a) $f(x) = x^3 - 3x$

$\frac{\partial f}{\partial x} = 3x^2 - 3 = 0$ to find max/min

$3(x^2-1) = 3(x+1)(x-1)$ $x = \pm 1$

so: $f(1) = 1-3 = -2$ and $f(-1) = -1+3 = 2$

$c = 2$ $c = -2$

(c) $f(x) = 2x^3 + 9x^2 + 12x + 7$

$\frac{\partial f}{\partial x} = 6x^2 + 18x + 12$

$= 6(x^2+3x+2) = 6(x+1)(x+2)$, $x = -1, -2$

$f(-1) = -2 + 9 - 12 + 7$

$= 2$

$c = -2$

or

$f(-2) = -2(8) + 9(4) - 24 + 7$

$= -16 + 36 - 24 + 7$

$= 3$

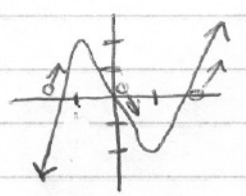
$c = -3$

2. I. $\lim_{x \to \infty} f(x,y) = \infty$ & $\lim_{x \to -\infty} f(x,y) = -\infty$

Consider assertion: for such a function the number of equilibrium points with $\frac{\partial f}{\partial x} > 0$ − $\frac{\partial f}{\partial x} < 0$ = 1

is it true for:

(a) $f(x,y) = x^3 - 3x + y$



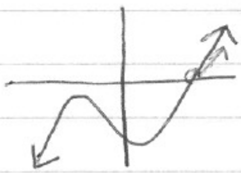
$y = 0$

$\frac{\partial f}{\partial x} > 0 = 2$

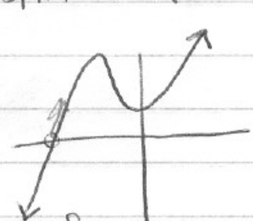
$- \frac{\partial f}{\partial x} < 0 = 1$

1 ✓

same is true for all shifts (all values of y)

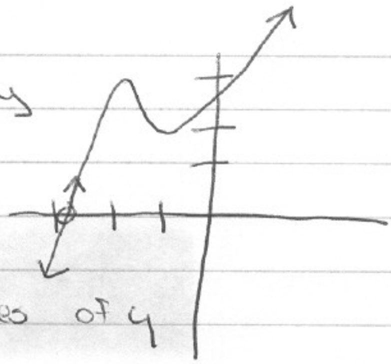


or



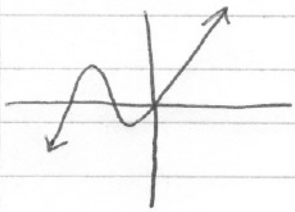
1 eq pt p $\frac{\partial f}{\partial x} > 0$

2 (c). $f(x) = 2x^3 + 9x^2 + 12x + 7 + y$



1 eq pt w/ $\frac{\partial f}{\partial x} > 0$

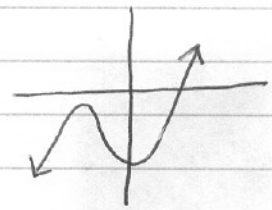
same is true for all values of y



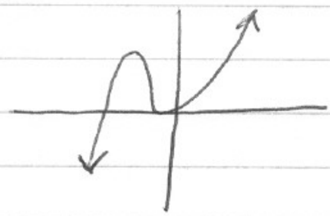
$\frac{\partial f}{\partial x} > 0 = 2$

$- \frac{\partial f}{\partial x} < 0 = 1$

1

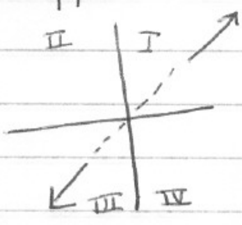


$\partial f/\partial x > 0 = 1$



$\frac{\partial f}{\partial x} > 0 = 1$

2. II The assertion will always hold for functions that $\to -\infty$ as $x \to -\infty$ & ∞ as $x \to \infty$ because the funtion must Begin by ~~de~~ in creasing in quadrant & end by increasing in quadrant I regardless of what happens in between.



There fore, ~~No matter~~ the function must increase over the x axis once. The function may then decrease over the axis, but in order to fit the given condition, it must then increase over the x axis an additional time.