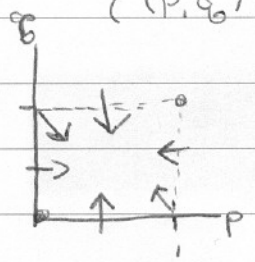
Ch. 23

1a, e, & 2a

1. $\frac{\partial}{\partial s}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} f(p,q) \\ g(p,q) \end{pmatrix}$ $\{(p,q) : 0 \le p \le 1,\ 0 \le q \le 1\}$



(a) $f(p,q) = q - p$

$g(p,q) = p - q$

Sides

$p=1$ $\frac{\partial p}{\partial s} = q - p = q - 1$; $q \le 1$ $\therefore$ $\frac{\partial p}{\partial s} \le 0$

$p=0$ $\frac{\partial p}{\partial s} = q$ $\therefore$ $q \ge 1$ $\therefore$ $\frac{\partial p}{\partial s} \ge 0$

$q=0$ $\frac{\partial q}{\partial s} = p - q = p$ $\therefore$ $\frac{\partial p}{\partial s} \ge 0$

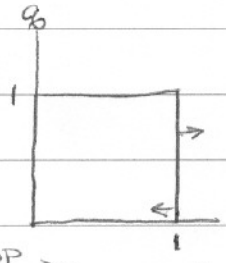
$q=1$ $\frac{\partial q}{\partial s} = p - 1$; $p \le 1$ $\therefore$ $\frac{\partial q}{\partial s} \le 0$

Corners		
(0,0)	$\frac{\partial p}{\partial s} = 0$	$\frac{\partial q}{\partial s} = 0$
(1,0)	$\frac{\partial p}{\partial s} = -1$	$\frac{\partial q}{\partial s} = 1$
(1,1)	$\frac{\partial p}{\partial s} = 0$	$\frac{\partial q}{\partial s} = 0$
(0,1)	$\frac{\partial p}{\partial s} = 1$	$\frac{\partial q}{\partial s} = -1$

Yes a is a basin of attraction.

(c) $f(p,q) = p(q - \frac{1}{2})$ $p=1$ $\frac{dp}{ds} = p(q - \frac{1}{2})$

$g(p,q) = -q$

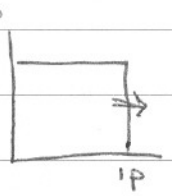


Not a basin of attraction because when $q > 1/2$ & $p=1$ $\frac{dp}{ds} > 0$ & all

exit.

(e) $f(p,q) = qp(p - \frac{1}{2})$ $p=1$ $\frac{dp}{ds} = q(+\frac{1}{2}) = > 0$

$g(p,q) = pq - 1$ $\therefore$ Not a basin of Attraction.



2. (a) $\begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}$ $Tr = 4 > 0$ $Det = 3 - 3 = 1 > 0$ Repelling

(c) $\begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix}$ $Tr = -2 < 0$ $Det = -3 + 4 = 1 > 0$ · Stable