Math 19

PS # 15 key

Chap. 20 & 21

Ch. 20

1) (a) advection: blood is the ambient fluid

(b) diffusion: assume mosquitoes move randomly

(c) advection: gulf stream current is ambient fluid (you might see some diffusion, but advection will probably be the major variable)

(d) diffussion: size limits contribution of ambient fluid

2. (a) $\lambda g = \frac{\partial^2}{\partial x^2} g - (x^2+1)g$ let $(x^2+1) = b$ when $0 \leq x \leq 1$ $b > 0$

so: if maximum: $g > 0$, $\frac{\partial^2 g}{\partial x^2} < 0$.

$$\underbrace{\lambda g}_{>0} = \underbrace{\frac{\partial^2 g}{\partial x^2}}_{<0} \underbrace{-}_{<0} \underbrace{b}_{} \underbrace{g}_{>0} \quad \text{RHS} < 0 \therefore \lambda < 0$$

if minimum: $g < 0$, $\frac{\partial^2 g}{\partial x^2} > 0$

$$\underbrace{\lambda g}_{<0} = \underbrace{\frac{\partial^2 g}{\partial x^2}}_{>0} - \underbrace{b}_{<0} \underbrace{g}_{<0} \quad \text{RHS} > 0 \therefore \lambda < 0$$

No such pair (λ, g) s.t. $\lambda \geq 0$ & $g \not\equiv 0$ stable

(b) $\lambda g = \frac{\partial^2 g}{\partial x^2} + (x^2+1)g$

if maximum: $g > 0$, $g'' < 0$

$$\underbrace{\lambda g}_{>0} = \underbrace{g''}_{<0} + \underbrace{(x^2+1)}_{>0} \underbrace{g}_{>0}$$

RHS may be $>, <, = 0$

if minimum: $g < 0$, $g'' > 0$

$$\lambda g = \underbrace{g''}_{>0} + \underbrace{(x^2+1)}_{>0} \underbrace{g}_{<0}$$

RHS may be $+, -,$ or 0

UNABLE To Tell with max principle

(c) $\lambda g = \frac{\partial^2 g}{\partial x^2} + (x^2-1)g$ at $x=1$: $(x^2-1)=0$ else $(x^2-1)<0$ {for this range of x}

if max: $\underbrace{\lambda g}_{>0} = \underbrace{\frac{\partial^2 g}{\partial x^2}}_{<0} + \underbrace{(x^2-1)g}_{<0}$ RHS <0; $\lambda < 0$

if min: $\underbrace{\lambda g}_{<0} = \underbrace{g''}_{>0} + \underbrace{(x^2-1)g}_{>0}$ RHS >0; $\lambda < 0$

if $x=1$, max: $\underbrace{\lambda g}_{>0} = \underbrace{g''}_{<0}$ $\therefore \lambda < 0$ | $x=1$: min $\underbrace{\lambda g}_{<0} = \underbrace{g''}_{>0}$ $\therefore \lambda < 0$

No pair st. $\lambda \geq 0$ & $g \not\equiv 0$ stable!

(d) $\lambda g = \frac{\partial^2}{\partial x^2} g + (2x^2-1)g$ $\quad$ $2x^2-1$, changes sighs over test interval

Can't tell with max min principle because $(2x^2-1)$ may be positive @ max & negative @ min.

Ch. 21 $\quad u(t,x) = f(x-ct)$ let $x-ct = s$ so $u(t,x) = f(s)$

now $\frac{\partial}{\partial t} u = \frac{\partial f}{\partial t} =$ (by chain rule) $\frac{\partial f}{\partial s}\frac{\partial s}{\partial t} = -c\frac{\partial f}{\partial s}$

similarly $\frac{\partial u}{\partial x} = \frac{\partial f}{\partial x} = \frac{\partial f}{\partial s}\frac{\partial s}{\partial x} = (1)\frac{\partial f}{\partial s}$

and $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 f}{\partial s^2}$

Now the problems become substitution

1. (a) $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial t^2} + 2(u - \sin(u)) \Rightarrow -c\frac{\partial f}{\partial s} = 3\frac{\partial^2 f}{\partial s^2} + 2(f - \sin(f)))$

(b) $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial u}{\partial x} + u^3 \Longrightarrow -c\frac{\partial f}{\partial s} = 4\frac{\partial^2 f}{\partial s^2} - 6\frac{\partial f}{\partial s} + f^3$

(c) $\frac{\partial u}{\partial t} = -5\frac{\partial^2 u}{\partial x^2} + e^u \Rightarrow -c\frac{\partial f}{\partial s} = -5\frac{\partial^2 f}{\partial s^2} + e^f$ (there is a mistake in the book on this one)

(d) $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} - 25u^2 \Longrightarrow c^2\frac{\partial^2 f}{\partial s^2} = 4\frac{\partial^2 f}{\partial s^2} - 25f^2$

$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial s}(-c)\right) = -c\left(\frac{\partial}{\partial s}\left(\frac{\partial f}{\partial s}\right)\right)\cdot\left(\frac{\partial s}{\partial t}\right) = c^2\frac{\partial^2 f}{\partial s^2}$

2. (a) for 1(a) let $\frac{\partial f}{\partial s} = p$ & $\frac{\partial^2 f}{\partial s^2} = \frac{dp}{ds}$

$-c\frac{\partial f}{\partial s} = 3\frac{\partial^2 f}{\partial s^2} + 2(f - \sin(f)) \Rightarrow -cp = 3\frac{\partial p}{\partial s} + 2(f - \sin(f)))$

$p = \frac{df}{ds}$

for 1(c)

$-c\frac{\partial f}{\partial s} = -5\frac{\partial^2 f}{\partial s^2} + e^f \Longrightarrow -cp = -5\frac{\partial p}{\partial s} + e^f$

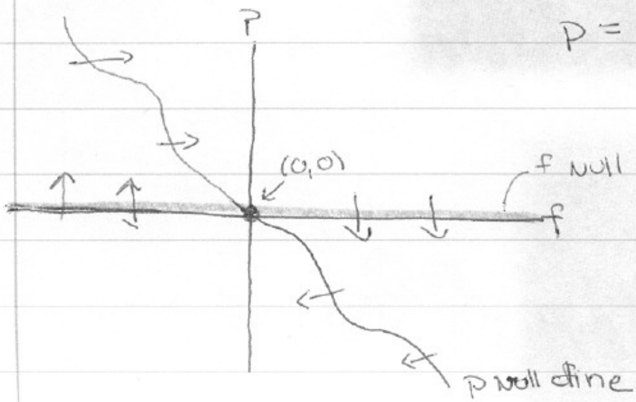
2(b) for 1(a) $\frac{dp}{ds} = \frac{-cp - 2(f - \sin(f))}{3}$

$\frac{df}{ds} = p$

f Null cline $\frac{df}{ds} = 0 = p$

p Null Cline $\frac{dp}{ds} = 0 = -cp - 2(f - \sin(f))$

$p = \frac{-2}{c}f + \frac{2}{c}\sin(f)$



$H(0,0) = \begin{pmatrix} \frac{-c}{3} & -\frac{2}{3} - \frac{2}{3}\cos(0) \\ 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} -c/3 & -4/3 \\ 1 & 0 \end{pmatrix}$

$tr = -c/3 < 0$ ✓

$det = -(-4/3) > 0$ ✓

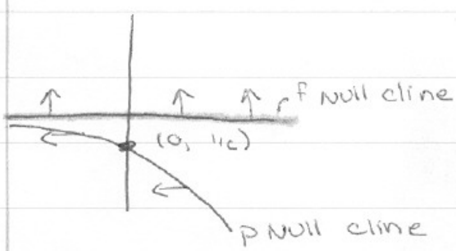
(0,0) is stable equilibrium

for 1(c) $\frac{dp}{ds} = \frac{cp + e^f}{5}$

$\frac{df}{ds} = p$

f Null cline $\frac{df}{ds} = 0 = p$

p Null cline $\frac{dp}{ds} = 0 = cp + e^f$ ∴ $p = -e^f/c$



No intersection ∴ No equilibrium pt.

3. Does Not account for:

Mouse food supply, weather limitations, seasonal changes, Mating patters, human control of mouse population, development of resistance, etc.