Ch. 18 Key

1. (7) $\cdot\ \mu U_{\varepsilon xx} + f(U_\varepsilon) = 0$ (8) $U_{\varepsilon x}(0) = U_{\varepsilon x}(L) = 0$

$f(U_\varepsilon) = aU_\varepsilon$

so if we let $U_\varepsilon = h$

$\mu \frac{\partial^2 h}{\partial x^2} + ah = 0$ we know that $\mu > 0$ & $a > 0$

$\frac{\partial^2 h}{\partial x^2} = \frac{-a}{\mu} h = ch$ ∴ $c < 0$

So: $h(x) = \alpha \cos\left(\sqrt{\frac{a}{\mu}}\, x\right) + \beta \sin\left(\sqrt{a/\mu}\, x\right)$

$h'(x) = \sqrt{\frac{a}{\mu}}\left(\alpha \overset{-\sin}{\cancel{\cos}}\left(\sqrt{\frac{a}{\mu}}\, x\right) + \beta \overset{\cos}{\cancel{\sin}}\left(\sqrt{\frac{a}{\mu}}\, x\right)\right)$

$h'(0) = \overset{\beta}{\cancel{\alpha}} \sqrt{\frac{a}{\mu}} = 0$ ∴ $\overset{\beta}{\cancel{\alpha}} = 0$

$h'(L) = \overset{-\alpha}{\cancel{\beta}} \sqrt{\frac{a}{\mu}} \sin\left(\sqrt{\frac{a}{\mu}}\, L\right) = 0$

either $\overset{\alpha}{\cancel{\beta}} = 0$ in which case => No solution

or $\sin\left(\sqrt{\frac{a}{\mu}}\, L\right) = 0$ & $\sqrt{\frac{a}{\mu}}\, L = n\pi$

$a = \left(\frac{n^2\pi^2}{L^2}\right)\mu$ as given!

2. $f(U_\varepsilon) = -aU_\varepsilon$

$\frac{\partial^2 h}{\partial x^2} = \frac{a}{\mu} h = ch$ so $c > 0$ b/c a & $\mu > 0$

So: $h(x) = \alpha e^{\sqrt{\frac{a}{\mu}}\, x} + \beta e^{-\sqrt{a/\mu}\, x}$

$h'(x) = \sqrt{\frac{a}{\mu}}\left(\alpha e^{\sqrt{a/\mu}\, x} - \beta e^{-\sqrt{a/\mu}\, x}\right)$

$h'(0) = \sqrt{\frac{a}{\mu}}(\alpha - \beta) = 0$ so. $\alpha = \beta$

$h'(L) = \sqrt{\frac{a}{\mu}}\, \alpha \left(e^{\sqrt{a/\mu}\, L} - e^{-\sqrt{a/\mu}\, L}\right)$ ∴ $\alpha = 0$

No solutions!

3. Achieving this exact ration is HIGHLEY unlikely ∴ you would not expect tigers to have stripes

5. (a) $f(u) = \sin(u)$ $\quad u_e(x) = \cos(2x)$ $\quad f'(u) = \cos(u)$

$f(u_e(x)) = \sin(\cos(2x))$

$f'(u_e(x)) = \cancel{2\sin(2x)\cos(\cos(2x))}$ $\cos(\cos(2x))$

(c) $f(u) = (1+u^2)$ $\quad u_e(x) = (1+x^2)$ $\quad f'(u) = 2u$

$f(u_e(x)) = (1 + (1+x^2)^2) = \cancel{1+1+2x^2+x^4}$

$f'(u_e(x)) = \cancel{2 \cdot 2x\,(1+x^2)}$ $2+2x^2$