Ch. 16 Key

1. at $u(1,x)$ the equation becomes

$$u(1,x) = Re^{-x^2/4\mu t}$$

the value of $u(1,0)$ can be determined from the graph. say the value is $u(1,0) = a$

then $u(1,0) = Re^0 = R = a$

so, $u(1,x) = ae^{-x^2/4\mu t}$

Now, from the graph, find the value of $u(1,1)$ and call this b

so, $u(1,1) = b = ae^{-1/4\mu t}$

solve for μ:

$$b = ae^{-1/4\mu t}$$

$$\frac{b}{a} = e^{-1/4\mu t}$$

$$\ln\frac{b}{a} = \frac{-1}{4\mu t}$$

$$\mu = \frac{-1}{4\ln(b/a)}$$

the same method applies for $u(2,x)$

at $x=0$; $u(2,0) = R\frac{1}{\sqrt{2}}e^0 = R\frac{1}{\sqrt{2}} = c$

so $R = \sqrt{2}\,c$ where c is the value from the graph of $u(2,0)$

at $x=1$; $u(2,1) = \sqrt{2}\,c\,\frac{1}{\sqrt{2}}e^{-\frac{1}{4\mu 2}} = ce^{-1/8\mu} = d$

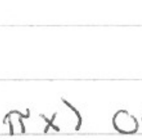
solve for μ: $ce^{-1/8\mu} = d$

$$e^{-1/8\mu} = d/c$$

$$-\frac{1}{8\mu} = \ln\left(\frac{d}{c}\right)$$

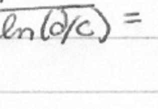
$$-\frac{1}{8\ln(d/c)} = \mu$$

3(a). $\sin(\pi x)$ $0 \le x \le 1$



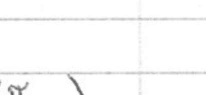
Not Negative

$\sin(3\pi x)$ $0 \le x \le 1$



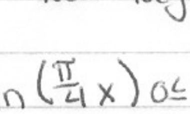
Negative

$\cos\left(\frac{\pi}{2}x\right)$ $0 \le x \le 1$



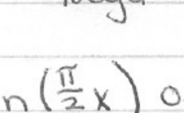
Not Negative

* (c) $\sin\left(\frac{\pi}{4}x\right)$ $0 \le x \le 3/2$



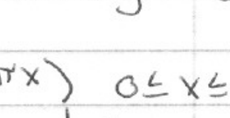
Not Negative

~~cos~~ $\sin\left(\frac{\pi}{2}x\right)$ $0 \le x \le 3/2$



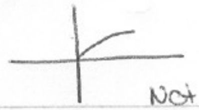
~~Not~~ Negative

$\sin(\pi x)$ $0 \le x \le 3/2$



Negative

c. $\sin\left(\frac{\pi x}{2}\right) \quad 0 \le x \le 3/2$



Not negative

4. find α & β when ~~B(x)~~ $B(x)=0$ at $x=0,1$

(a) $B(x) = \alpha e^{5x} + \beta e^{-5x}$

$B(0) = \alpha + \beta = 0$ $\qquad$ $B(1) = \alpha e^{5} + \beta e^{-5} = 0$

$\alpha = -\beta$ $\qquad$ $\alpha e^{5} - \alpha e^{-5} = 0$

$\alpha(e^{5} - e^{-5}) = 0$

$\alpha = 0$

$\boxed{\alpha = \beta = 0}$

(b) $B(x) = \alpha\cos(\pi x) + \beta\sin(\pi x)$

$B(0) = \alpha = 0$

$B(1) = \beta\sin(\pi) = \beta(0) \Rightarrow \beta =$ any thing

$\boxed{\alpha = 0, \ \beta = \text{any real number}}$

(c) $B(x) = \alpha e^{3\pi x} + \beta e^{-3\pi x}$

$B(0) = \alpha + \beta = 0$ $\qquad$ $B(1) = \alpha e^{3\pi} + \beta e^{-3\pi} = 0$

$\alpha = -\beta$ $\qquad$ $\alpha e^{3\pi} - \alpha e^{-3\pi} = 0$

$\alpha(e^{3\pi} - e^{-3\pi}) = 0$

$\alpha = 0$

$\boxed{\alpha = \beta = 0}$