Ch. 15 Key

1. (a) $u(t,x) = 2\frac{1}{t^{1/2}} e^{-x^2/4\mu t} + \frac{1}{t^{1/2}} e^{-(x-1)^2/4\mu t}$

$$\frac{\partial u}{\partial t} = -t^{-3/2} e^{-x^2/4\mu t} + \frac{2}{t^{1/2}} e^{-x^2/4\mu t}\left(\frac{x^2}{4\mu t^2}\right) - \frac{1}{2} t^{-3/2} e^{-(x-1)^2/4\mu t} + t^{-1/2} e^{-(x-1)^2/4\mu t}\left(\frac{(x-1)^2}{4\mu t^2}\right)$$

$$u_x = 2\frac{1}{t^{1/2}}\left(\frac{-2x}{4\mu t}\right) e^{-x^2/4\mu t} + \frac{1}{t^{1/2}}(-2)\left(\frac{x-1}{4\mu t}\right) e^{-(x-1)^2/4\mu t}$$

$$u_{xx} = 2t^{-1/2} e^{-x^2/4\mu t}\frac{x^2}{4\mu^2 t^2} + t^{-1/2} e^{-\frac{(x-1)^2}{4\mu t}}\left(\frac{(x-1)^2}{4\mu^2 t^2}\right) - \frac{1}{4\mu} t^{-1/2} e^{-x^2/4\mu t} - \frac{1}{2\mu t} t^{-1/2} e^{-\frac{(x-1)^2}{4\mu t}}$$

$$4u_{xx} = 2t^{-1/2} e^{-x^2/4\mu t}\frac{x^2}{4\mu t^2} + t^{-1/2} e^{-\frac{(x-1)^2}{4\mu t}}\left(\frac{(x-1)^2}{4\mu t^2}\right) - \frac{1}{t} t^{-1/2} e^{-x^2/4\mu t} - \frac{1}{2t} t^{-1/2} e^{-\frac{(x-1)^2}{4\mu t}}$$

(c) $u(t,x) = 2\frac{1}{t^{1/2}} e^{-x^2/4\mu t} + 3\frac{1}{t^{1/2}} e^{-(x-1)^2/4\mu t}$

$$u_t = -t^{-3/2} e^{-x^2/4\mu t} + \frac{x^2}{2\mu} t^{-5/2} e^{-x^2/4\mu t} - \frac{3}{2} t^{-3/2} e^{-(x-1)^2/4\mu t} + \frac{3}{4}\frac{(x-1)^2}{\mu} t^{-5/2} e^{-(x-1)^2/4\mu t}$$

$$u_x = 2\frac{1}{t^{1/2}}\left(\frac{-2x}{4\mu t}\right) e^{-x^2/4\mu t} + 3\left(\frac{1}{t^{1/2}}\right)\left(\frac{-2(x-1)}{4\mu t}\right) e^{-(x-1)^2/4\mu t}$$

$$u_{xx} = \frac{x^2}{2\mu^2} t^{-5/2} e^{-x^2/4\mu t} + \frac{3}{4\mu^2}(x-1)^2 t^{-5/2} e^{-(x-1)^2/4\mu t} - t^{-3/2} e^{-x^2/4\mu t}\frac{1}{\mu} - \mu\frac{3}{2} t^{-3/2} e^{-(x-1)^2/4\mu t}$$

2.(a)

$$u(t,x) = 2e^{\lambda t} e^{x(\lambda/\nu)^{1/2}} + 3e^{\lambda t} e^{-x(\lambda/\mu)^{1/2}}$$

$$u_t = \lambda 2e^{\lambda t} e^{x(\lambda/\nu)^{1/2}} + \lambda 3e^{\lambda t} e^{-x(\lambda/\mu)^{1/2}} = \lambda u$$

$$u_x = \left(\frac{\lambda}{\nu}\right)^{1/2} 2e^{\lambda t} e^{x(\lambda/\nu)^{1/2}} + -\left(\frac{\lambda}{\mu}\right)^{1/2} 3e^{\lambda t} e^{-x(\lambda/\mu)^{1/2}}$$

$$u_{xx} = \frac{\lambda}{\mu} 2e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} + \frac{\lambda}{\mu} 3e^{\lambda t} e^{-x(\lambda/\mu)^{1/2}} = \frac{\lambda}{\mu} u$$

$$u_t = \mu u_{xx}$$

$$\lambda u = \mu \frac{\lambda}{\mu} u = \lambda u \quad \checkmark$$

2.(c)

$$u(t,x) = 2e^{\lambda t} e^{x(\lambda/\nu)^{1/2}} + 1 - 5x$$

$$u_t = \lambda 2e^{\lambda t} e^{x(\lambda/\nu)^{1/2}}$$

$$u_x = \left(\frac{\lambda}{\nu}\right) 2e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} - 5$$

$$u_{xx} = \frac{\lambda}{\mu} 2e^{\lambda t} e^{x(\lambda/\mu)^{1/2}}$$

$$u_t = \mu u_{xx}$$

$$\lambda 2e^{\lambda t} e^{x(\lambda/\nu)^{1/2}} = \mu \frac{\lambda}{\mu} 2e^{\lambda t} e^{x(\lambda/\nu)^{1/2}} \quad \checkmark$$