Problem Set #1 KEY Math 19

pg. 43 - 44

#'s: 1, 2, 4 a, b, c, d, & 5

1. (a) $\frac{dy}{dt} = 5y$

$\int \frac{1}{y} dy = \int 5 dt$

$\ln y = 5t + C$

$y = e^{5t+C} = Ce^{5t}$ because $e^C = \text{Constant}$

(b) $\frac{dy}{dt} = -3y$

$\ln y = -3t + C$

$y = e^{-3t+C} = Ce^{-3t}$

(c) $\frac{dy}{dt} = 12y$

$\ln y = 12t + C$

$y = e^{12t+C} = Ce^{12t}$

(d) $\frac{dy}{dt} = -1.5y$

$\ln y = -1.5t + C$

$y = e^{-1.5t+C} = Ce^{-1.5}$

2. (a) $y(0) = 1$

$y = Ce^{5t}$	$y = Ce^{-3t}$	$y = Ce^{12t}$	$y = Ce^{-1.5t}$
$1 = Ce^{0}$	$1 = Ce^{0}$	$1 = Ce^{0}$	$1 = Ce^{0}$
$\frac{1}{e^0} = C = 1$	$\frac{1}{e^0} = C = 1$	$\frac{1}{e^0} = C = 1$	$\frac{1}{e^0} = C = 1$
$y = e^{5t}$	$y = e^{-3t}$	$y = e^{12t}$	$y = e^{-1.5t}$

(b) $y(1) = 1$

$y = Ce^{5t}$	$y = Ce^{-3t}$	$y = Ce^{12t}$	$y = Ce^{-1.5t}$
$1 = Ce^{5}$	$1 = Ce^{-3}$	$1 = Ce^{12}$	$1 = Ce^{-1.5}$
$\frac{1}{e^5} = C = e^{-5}$	$\frac{1}{e^{-3}} = C = e^{3}$	$\frac{1}{e^{12}} = C = e^{-12}$	$\frac{1}{e^{-1.5}} = C = e^{1.5}$
$y = e^{-5}e^{5t}$	$y = e^{3}e^{-3t}$	$y = e^{-12}e^{12t}$	$y = e^{1.5}e^{-1.5t}$

(c) $y(-1) = 1$

$y = Ce^{5t}$	$y = Ce^{-3t}$	$y = Ce^{12t}$	$y = Ce^{-1.5t}$
$1 = Ce^{-5}$	$1 = Ce^{3}$	$1 = Ce^{-12}$	$1 = Ce^{1.5}$
$\frac{1}{e^{-5}} = C = e^{5}$	$\frac{1}{e^{3}} = C = e^{-3}$	$\frac{1}{e^{-12}} = C = e^{12}$	$\frac{1}{e^{1.5}} = C = e^{-1.5}$
$y = e^{5}e^{5t}$	$y = e^{-3}e^{-3t}$	$y = e^{12}e^{12t}$	$y = e^{-1.5}e^{-1.5t}$

Problem Set #1 KEY

(d) $y(-1) = -1$

$y = Ce^{5t}$
$-1 = Ce^{-5}$
$\frac{-1}{e^{-5}} = C = -e^{5}$
$y = -e^{5}e^{5t}$

$y = Ce^{-3t}$
$-1 = Ce^{3}$
$\frac{-1}{e^{3}} = C = -e^{-3}$
$y = -e^{-3}e^{-3t}$

$y = Ce^{12t}$
$-1 = Ce^{-12}$
$\frac{-1}{e^{-12}} = C = -e^{12}$
$y = -e^{12}e^{12t}$

$y = Ce$
$-1 = Ce$
$\frac{-1}{e^{1.5}} = C$
$y = -e^{-}$

. First Order Taylor expansion at $x_0 = 0$:

$g(x) = f(0) + f'(0)x$

a. $f(x) = \sin(x) \Rightarrow f(0) = \sin(0) = 0$

$f'(x) = \cos(x) \Rightarrow f'(0) = \cos(0) = 1$

$g(x) = \sin(0) + (\cos(0))x = 0 + x = x$

b. $f(x) = e^{x} \quad f(0) = e^{0} = 1$

$f'(x) = e^{x} \quad f'(0) = e^{0} = 1$

$g(x) = e^{0} + (e^{0})x = 1 + x$

c. $f(x) = \frac{x}{1+x^2} \quad f(0) = \frac{0}{1} = 0$

$f'(x) = \frac{(1+x^2) - x(2x)}{(1+x^2)^2} \quad f'(0) = \frac{1-0}{1} = 1$

$g(x) = 0 + 1x = x$

d. $f(x) = e^{x}\sin(x) \quad f(0) = e^{0}\sin(0) = 1 \times 0 = 0$

$f'(x) = e^{x}\cos(x) + e^{x}\sin(x) \quad f'(0) = e^{0}\cos(0) + e^{0}\sin(0) =$

$g(x) = e^{0}\sin(0) + (e^{0}\cos(0) + e^{0}\sin(0))x = 0 + 1x =$

5. Known: 1.) An individual gives birth 4 times a day on average

$\therefore$ birth rate = 4 / day

2.) An individual dies after 1 day on average

$\therefore$ death rate = 1 / day

$\frac{dP}{dt} = aP$ where $a =$ (birth rate − death rate)

so: $\frac{dP}{dt} = (4-1)P = 3P$

$$\int \frac{1}{P}\,dP = \int 3\,dt$$

$$\ln P = 3t + C$$

$$P = e^{3t+C} = Ce^{3t}$$

$P(0) = 1000$

so: $P = Ce^{0} = 1000$

$$C = 1000$$

$$P = 1000e^{3t}$$