# Homework 9

## Chapter 13

1. $u(t,x) = e^{-2t}\sin(x-3t)$

$\frac{d}{dt}u(t,x) = -2e^{-2t}\cdot(\sin(x-3t)) + e^{-2t}\cos(x-3t)\cdot -3$

$\frac{d}{dx}u(t,x) = e^{-2t}\cdot\cos(x-3t)$

the equality $\frac{d}{dt}u(t,x) = -3\frac{d}{dx}u(t,x) - 2u(t,x)$ does hold

3. $u(t,x) = e^{-2t}\, e^{-2(x-3t)/3} = e^{-2x/3}$ (not $e^{-2x}$, as stated in the book)

$\frac{d}{dt}u(t,x) = 0 \quad \frac{d}{dx}u(t,x) = -\frac{2}{3}e^{-2x/3}$

the equality does hold

5. $u(t,x) = e^{-2t}(1+(x-3t)^2)^{-1}$

$\frac{d}{dt}u(t,x) = -2e^{-2t}(1+(x-3t)^2)^{-1} + e^{-2t}(1+(x-3t)^2)^{-2}\cdot 2(x-3t)\cdot 3$

$\frac{d}{dx}u(t,x) = -e^{-2t}(1+(x-3t)^2)^{-2}\cdot 2(x-3t)$

the equality does hold

6. when $r=2$, find the solution to

$\frac{d}{dt}u(t,x) = -3\frac{d}{dx}u(x,t) - 2u(t,x)$ with $u(0,x) = \cos x$

$u(t,x) = e^{-rt}\cdot F(x-3t) \quad u(0,x) = \cos x \implies F(x) = \cos x$

so $u(t,x) = e^{-2t}\cos(x-3t)$

7. $u(0,x) = e^{-4x}$

$u(t,x) = e^{-rt}\cdot F(x-3t) \implies F(x) = e^{-4x}$, so

$u(t,x) = e^{-2t}\cdot e^{-4(x-3t)} = e^{-4x+10t}$

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8. $u(0,x) = (1+x^2)^{-1} \Rightarrow f(x) = (1+x^2)^{-1}$

$u(x,t) = e^{-2t} \cdot f(x-3t)$

$u(x,t) = e^{-2t}(1+(x-3t)^2)^{-1}$

## Chapter 14

1. $f(t,x) = \sin(x-3t)$

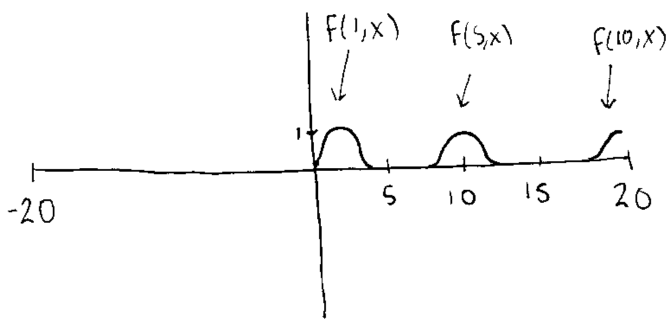
$\frac{df}{dt} = -3\cos(x-3t) \quad \frac{df}{dx} = \cos(x-3t)$

3. $f(t,x) = t^{-1/2} e^{-x^2/t}$

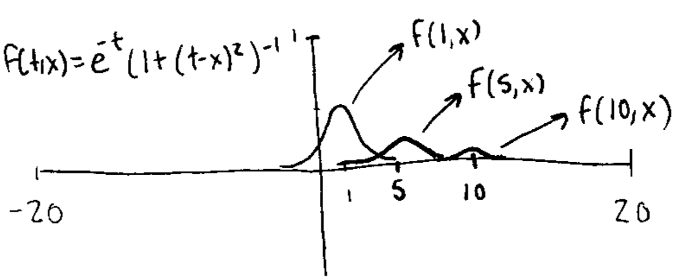
$\frac{df}{dt} = -\frac{1}{2} t^{-3/2} \cdot e^{-x^2/t} + t^{-1/2} e^{-x^2/t} \cdot x^2 t^{-2}$

$\frac{df}{dx} = t^{-1/2} \cdot e^{-x^2/t} \cdot \frac{-2x}{t}$

5. $f(t,x) = e^{-(x-2t)^2}$




6. $f(t,x) = e^{-t}(1+(t-x)^2)^{-1}$



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8. $F(t,x) = e^{-(x-2t)^2}$ $\quad r=0,\ c=2$

$F(t,x) = e^{-t}\left(1+(t-x)^2\right)^{-1}$ $\quad r=1,\ c=1$

$F(t,x) = t^{-1/2}\, e^{-x^2/t}$ $\quad \mu = 1/4$

10. + 12.

To verify that these are solutions, use the chain rule and the fact that $\frac{d}{dx} e^x = e^x$ to take the partial derivatives

$\frac{d}{dt} u$ and $\frac{d^2 u}{dx^2}$.