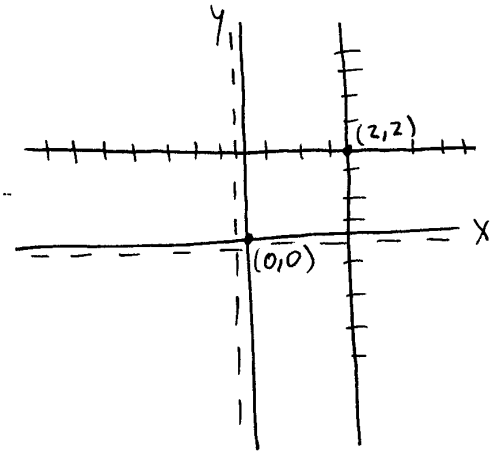
Homework 6

Chapter 9

1. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xy-2x \\ xy-2y \end{pmatrix}$ $\quad \frac{dx}{dt} = x(y-2) \Rightarrow \begin{matrix} x=0 \\ y=2 \end{matrix}\}$ x null clines $\quad \frac{dy}{dt} = y(x-2) \Rightarrow \begin{matrix} y=0 \\ x=2 \end{matrix}\}$ y null clines

equilibrium pts: (0,0), (2,2)



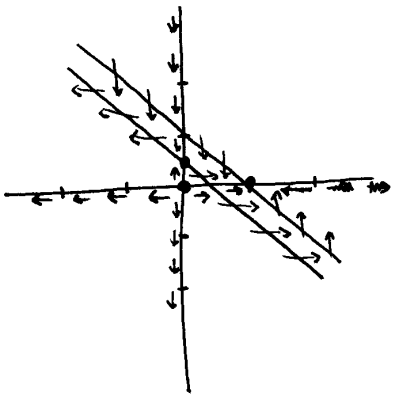
$$D = \begin{pmatrix} y-2 & x \\ y & x-2 \end{pmatrix}$$

$$D_{(0,0)} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \quad \begin{matrix} \det D = 4 > 0 \\ \operatorname{tr} D = -4 < 0 \end{matrix}\} \text{ stable at } (0,0)$$

$$D_{(2,2)} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \quad \begin{matrix} \det D = -4 < 0 \\ \operatorname{tr} D = 0 \end{matrix}\} \text{ unstable at } (2,2)$$

2. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-x^2-xy \\ y-2xy-2y^2 \end{pmatrix}$ $\quad \frac{dx}{dt} = x(1-x-y) \Rightarrow \begin{matrix} x=0 \\ y=1-x \end{matrix}\}$ x null clines

$\frac{dy}{dt} = y(1-2x-2y) \Rightarrow \begin{matrix} y=0 \\ y=\frac{1}{2}-x \end{matrix}\}$ y null clines



equilibrium points: $(0,\frac{1}{2}), (1,0), (0,0)$

$$D = \begin{pmatrix} 1-2x-y & -x \\ -2y & 1-2x-4y \end{pmatrix}$$

$$D_{(0,0)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{matrix} \det D = 1 > 0 \\ \operatorname{tr} D = 2 > 0 \end{matrix}\} \text{ unstable at } (0,0)$$

$$D_{(1,0)} = \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix} \quad \begin{matrix} \det D = 1 > 0 \\ \operatorname{tr} D = -2 < 0 \end{matrix}\} \text{ stable at } (1,0)$$

$$D_{(0,\frac{1}{2})} = \begin{pmatrix} \frac{1}{2} & 0 \\ -1 & -1 \end{pmatrix} \quad \begin{matrix} \det D = -\frac{1}{2} < 0 \\ \operatorname{tr} D = -\frac{1}{2} < 0 \end{matrix}\} \text{ unstable at } (0,\tfrac{1}{2})$$

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Chapter 9

4. a) $h(x,y) = xy^3 - x^2y^2$

$\frac{\partial h}{\partial x} = y^3 - 2xy^2 \qquad \frac{\partial h}{\partial y} = 3xy^2 - 2x^2y$

c) $\frac{\partial h}{\partial x} = \cos y \qquad \frac{\partial h}{\partial y} = -x\sin y$

5. $h(x,y) = xy^3 - x^2y^2$:

a) $h(x(t), y(t)) = \cos t \sin^3 t - \cos^2 t \sin^2 t$

b) $\frac{d}{dt} h(x(t), y(t)) = -\sin^4 t + 3\cos^2 t \sin^2 t + 2\sin^3 t \cos t - 2\cos^3 t \sin t$

c) $\frac{dh}{dt} = \frac{dh}{dx}\cdot\frac{dx}{dt} + \frac{dh}{dy}\cdot\frac{dy}{dt} = (y^3 - 2xy^2)(-\sin t) + (3xy^2 - 2x^2y)(\cos t) =$

$= -\sin^4 t + 2\cos t \sin^3 t + 3\cos^2 t \sin^2 t - 2\cos^3 t \sin t$

$h(x,y) = x\cos y - 1$

a) $h(x(t), y(t)) = \cos t \cos(\sin t) - 1$

b) $\frac{d}{dt} h(x(t), y(t)) = -\sin t \cdot (\cos(\sin t)) - \cos^2 t \cdot \sin(\sin t))$

c) $\frac{dh}{dt} = \frac{dh}{dx}\cdot\frac{dx}{dt} + \frac{dh}{dy}\cdot\frac{dy}{dt} = \cos y(-\sin t) + (-x\sin y)(\cos t)$

$= -\sin t \cos(\sin t) - \cos^2 t \cdot \sin(\sin t)$

Chapter 10

1. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - x^2 - 2xy \\ 2y - y^2 - 3xy \end{pmatrix} \qquad D = \begin{pmatrix} 1-2x-2y & -2x \\ -3y & 2-2y-3x \end{pmatrix}$

$D_{(0,0)} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ $\det D = 2 > 0$, $\operatorname{tr} D = 3 > 0$ } unstable at $(0,0)$

$D_{(1,0)} = \begin{pmatrix} -1 & -2 \\ 0 & -1 \end{pmatrix}$ $\det D = 1 > 0$, $\operatorname{tr} D = -2 < 0$ } stable at $(1,0)$

$D_{(0,2)} = \begin{pmatrix} -3 & 0 \\ -6 & -2 \end{pmatrix}$ $\det D = 6 > 0$, $\operatorname{tr} D = -5 < 0$ } stable at $(0,2)$

$D_{(.6,.2)} = \begin{pmatrix} -.6 & -1.2 \\ -.6 & -.2 \end{pmatrix}$ $\det D = -.6$, $\operatorname{tr} D = -.8$ } unstable at $(.6,.2)$

Homework 6

Chapter 10

3. $h = x^2y^3$

$\frac{\partial h}{\partial x} = 2xy^3 \quad \frac{\partial h}{\partial y} = 3x^2y^2$

$\frac{\partial^2 h}{\partial x \partial y} = 6xy^2 \quad \frac{\partial^2 h}{\partial y \partial x} = 6xy^2$

$\frac{\partial^2 h}{\partial x^2} = 2y^3 \quad \frac{\partial^2 h}{\partial y^2} = 6x^2y$

$h = x\cos(xy)$

$\frac{\partial h}{\partial x} = \cos(xy) - xy\sin(xy) \quad \frac{\partial h}{\partial y} = -x^2\sin(xy)$

$\frac{\partial^2 h}{\partial x \partial y} = -2x\sin(xy) - x^2y\cos(xy) \quad \frac{\partial^2 h}{\partial y^2} = -x^3\cos(xy)$

$\frac{\partial^2 h}{\partial x^2} = -2y\sin(xy) - xy^2\cos(xy)$

$\frac{\partial^2 h}{\partial y \partial x} = -2x\sin(xy) - x^2y\cos(xy)$

$h = \sin(x+y^2)$

$\frac{\partial h}{\partial x} = \cos(x+y^2) \quad \frac{\partial h}{\partial y} = 2y\cos(x+y^2)$

$\frac{\partial^2 h}{\partial x \partial y} = -2y\sin(x+y^2) \quad \frac{\partial^2 h}{\partial y \partial x} = -2y\sin(x+y^2)$

$\frac{\partial^2 h}{\partial x^2} = -\sin(x+y^2) \quad \frac{\partial^2 h}{\partial y^2} = 2\cos(x+y^2) - 2y\sin(x+y^2)\cdot 2y$

$h = xe^y$

$\frac{\partial h}{\partial x} = e^y \quad \frac{\partial h}{\partial y} = xe^y$

$\frac{\partial^2 h}{\partial x \partial y} = e^y \quad \frac{\partial^2 h}{\partial y \partial x} = e^y$

$\frac{\partial^2 h}{\partial x^2} = 0 \quad \frac{\partial^2 h}{\partial y^2} = xe^y$

Homework 6

Chapter 10

4. a) $h(x,y) = 1$, $0 \le x \le 1$ $\quad -1 \le y \le 2$

$$\int_0^1 \int_{-1}^2 h(x,y)\,dy\,dx = \int_0^1 \int_{-1}^2 dy\,dx = \int_0^1 y\Big|_{-1}^2\,dx = \int_0^1 3\,dx = 3$$

c) $$\int_0^1 \int_0^1 x+y\,dy\,dx = \int_0^1 \frac{y^2}{2}+xy\Big|_0^1\,dx = \int_0^1 \frac{1}{2}+x\,dx = \frac{x^2}{2}+\frac{x}{2}\Big|_0^1 = 1$$

e) $$\int_0^1 \int_0^1 \cos(xy)\,dy\,dx = \int_0^1 \frac{\sin(xy)}{x}\Big|_0^1\,dx = \int_0^1 \frac{\sin x}{x}\,dx \approx .946$$

(numerical integration on calculator)