Math 19 Homework 21

Chapter 27

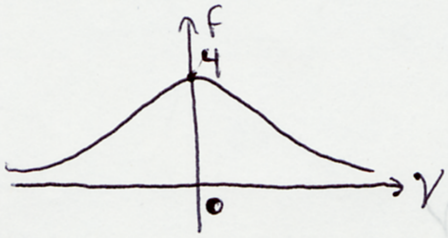
1. $m(t) = e^{-|t|}$

$$a(\nu) = \frac{1}{T}\int_{-\infty}^{\infty} e^{-|t|}\cos 2\pi\nu t\,dt = \frac{2}{T(1+4\pi^2\nu^2)}$$

$$b(\nu) = \frac{1}{T}\int_{-\infty}^{\infty} e^{-|t|}\sin 2\pi\nu t\,dt = 0$$

$\sigma = \frac{1}{T}$ $\quad f(\nu) = \dfrac{4}{T(1+4\pi^2\nu^2)^2}$

when $T=1$,



$f(\nu)$ reaches maximum at $\nu = 0$

2. $m(t) = e^{-|t+a|}$

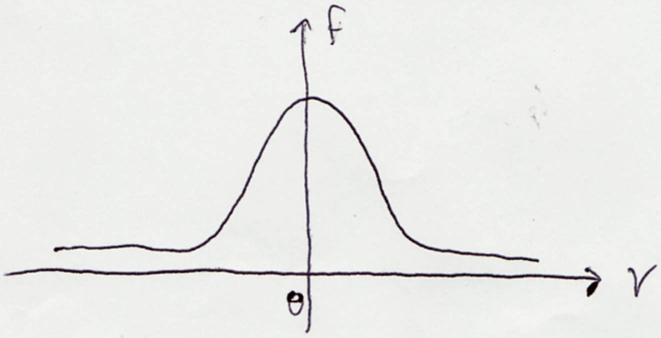
$$a(\nu) = \frac{1}{T}\int_{-\infty}^{\infty} e^{-|t+a|}\cos 2\pi\nu t\,dt$$

$$= \frac{2e^{a}\left(\cos(2\pi\nu a) + 2\pi\sin(2\pi\nu a)\right)}{T(1+4\pi^2\nu^2)}$$

$b(\nu) = 0$, $\sigma = 1/T$

$$F(\nu) = \frac{a(\nu)^2 + b(\nu)^2}{\sigma} = \frac{4e^{2a}\left(\cos(2\pi\nu a) + 2\pi\sin(2\pi\nu a)\right)^2}{T(1+4\pi^2\nu^2)}$$

when $T = a = 1$,



$f(\nu)$ reaches maximum at $\nu = 0$

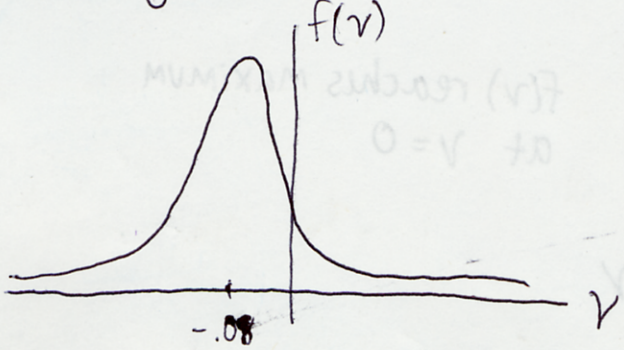
3. $m(t) = e^{-|t+1|} + e^{-|t|} + e^{-|t-1|}$

$$a(\nu) = \frac{2e\left(\cos(2\pi\nu) + 2\pi\sin(2\pi\nu)\right)}{\tau(1+4\pi^2\nu^2)} + \frac{2}{\tau(1+4\pi^2\nu^2)} + \frac{2e^{-1}\left(\cos(-2\pi\nu) + 2\pi\sin(-2\pi\nu)\right)}{\tau(1+4\pi^2\nu^2)}$$

$b(\nu) = 0 \qquad \sigma = \frac{1}{\tau}$

$f(\nu) = \dfrac{a(\nu)^2 + b(\nu)^2}{\sigma}$ when $\tau = 1$,

f



$f(\nu)$ reaches maximum at $\nu \approx .08$

4. See #3