Math 19 Homework 14

Chapter 25

1. $\frac{dx}{dt} = 3x + x^2 + 3 \quad x(0) = 0, \; x(t_1) = 1$

$\left.\frac{dx}{dt}\right|_{t=0} = 3 \quad \left.\frac{dx}{dt}\right|_{t=t_1} = 7 \quad \frac{dx}{dt} = f(x) \quad f_{min} \le \frac{dx}{dt} \le f_{max}$

$\int_0^{t_1} f_{min}\, dt \le x(t_1) - x(0) \le \int_0^{t_1} f_{max}\, dt$

$t_1 \cdot f_{min} \le 1 - 0 \le t_1 \cdot f_{max}$

$t_1 \le \frac{1}{f_{min}}, \quad t_1 \ge \frac{1}{f_{max}}$ $\quad f_{min}, f_{max}$ occur where $f'(x) = 3 + 2x = 0$ i.e. where $x = -3/2$, which is not between 0, 1.

So $f_{min} = 3$, $f_{max} = 7$, and $\frac{1}{7} \le t_1 \le \frac{1}{3}$

2. $\frac{dx}{dt} = x^3 + 5 = f(x)$

$f(0) = 5, \; f(1) = 6 \qquad \frac{1}{6} \le t_1 \le \frac{1}{5}$

$f'(x) = 3x^2 = 0 \Rightarrow x = 0$

5. $\frac{dx}{dt} = 2x - x^2 + 1 = f(x)$

$f(0) = 1, \; f(1) = 2 \qquad \frac{1}{2} \le t_1 \le 1$

$f'(x) = 2 - 2x = 0 \Rightarrow x = 1$

6. $\frac{dx}{dt} = 2x + x^2 + 3 = f(x)$

$f(0) = 3, \; f(1) = 6$

$f'(x) = 2 + 2x = 0 \Rightarrow x = -1$ (not between 0, 1)

$\frac{1}{6} \le t_1 \le \frac{1}{3}$