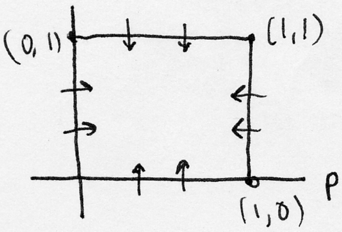
Chapter 23 Math 19 - Homework 18

p. 391-392

1. a) $f(p,q) = q-p \quad g(p,q) = p-q$

$S = \{(p,q): 0 \le p \le 1, 0 \le q \le 1\}$

is a basin of attraction:



$p=0 \to \frac{dp}{ds} = q-0 \ge 0$

$p=1 \to \frac{dp}{ds} = q-1 \le 0$

$q=0 \to \frac{dq}{ds} = p-0 \ge 0$

$q=1 \to \frac{dq}{ds} = p-1 \le 0$

c) $f(p,q) = p(q-\frac{1}{2}) \quad g(p,q) = -q$

when $p=1$, $\frac{dp}{ds} = q-\frac{1}{2}$ which can be positive

S is not a basin of attraction

e) $f(p,q) = qp(p-\frac{1}{2}) \quad g(p,q) = pq-1$

when $q=0$, $\frac{dq}{ds} = -1$

S is not a basin of attraction

2. a) $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ $\det A = 1$, $\operatorname{tr} A = 4$ $\Rightarrow$ repelling

c) $A = \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix}$ $\det A = 1$, $\operatorname{tr} A = -2$ $\Rightarrow$ stable

p. 410-411

1. a) $f(x) = x^3 - 3x$

$\frac{df}{dx} = 3x^2 - 3 = 0, \quad x = \pm 1$

$f(1) = -2, \ f(1) = 2$ so $\boxed{C = \pm 2}$

c) $f(x) = 2x^3 + 9x^2 + 12x + 7$

$\frac{df}{dx} = 6x^2 + 18x + 12 = 0 \quad x = -2, -1$

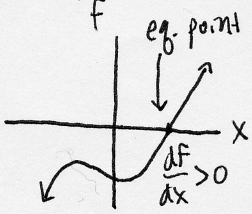
$f(-2) = 3, \ f(-1) = 2$ so $\boxed{C = -3, -2}$

p. 411

2. a) $f(x,y) = x^3 - 3x + y$

$\frac{df}{dx} = 3x^2 - 3 = 0 \quad x = \pm 1, \ \boxed{C = \pm 2}$

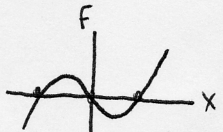
so for $y < -2$, have



positive negative

1 - 0 = 1

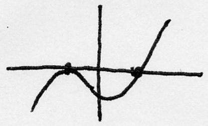
for $-2 < y < 2$, have



positive - negative

2 - 1 = 1

for $y = -2$, have



positive - negative

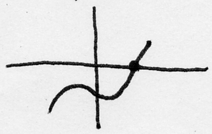
1 - 0 = 1

analagous cases for $y = 2, \ y > 2$

c) $f(x,y) = 2x^3 + 9x^2 + 12x + 7 + y$

$\boxed{C = -3, -2}$

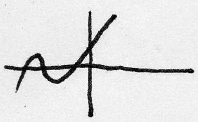
for $y < -3$, have



positive - negative

1 - 0 = 1

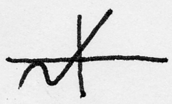
for $-3 < y < -2$, have



positive - negative

2 - 1 = 1

for $y = -3$, have



positive - negative

1 - 0 = 0

analagous for $y = -2, \ y > -2$

The assertion always holds because the first and last equilibrium points (for most negative + most positive x) must have $\frac{df}{dx} > 0$ or $\frac{df}{dx} = 0$, and the signs of $\frac{df}{dx}$ alternates over the eq. points.